Graph-based Total Variation for Tomographic Image Reconstruction

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***Abstract*—In this short abstract we introduce a novel method for tomographic reconstructions from low-dose data which is usually noisy and has missing information. Our method is a generalization of sparsity exploiting image reconstruction methods which employ Total Variation (TV) as an additional sparsifying transform. Similar to state-of-the-art Non-local TV (NLTV) method our proposed method goes beyond spatial similarity between different regions of an image being reconstructed by establishing a connection between similar regions in the image regardless of spatial distance. However, it involves updating the graph prior during every iteration and is computationally more efficient as compared to adaptive NLTV.**

Let $S \in \Re^{p\times q}$ be the sinogram corresponding to the collected tomographic projections of the sample $X \in \Re^{n\times n}$ being imaged, in a typical computed or electron tomography setup. Let p denote the number of rays passing through X and q the number of angular variations at which X has been imaged. Let $b \in \Re^{pq}$ be the vectorized measurements or projections ($b = vec(S)$), where $vec(\cdot)$ denotes the vectorization operation and $A \in \Re^{pq\times n^2}$ be the sparse projection operator. Then, the goal of a tomographic reconstruction method is to recover the vectorized sample $x = vec(X)$ from the projections b.

Sparsity exploiting methods, such as Compressed Sensing (CS) have been frequently used with Total Variation (TV)-based regularization CSTV [1] to efficiently solve such ill-posed problems. Recently, non-local TV (NLTV) [2], which exploits pairwise similarity between the different regions of the sample by constructing a sparse or dense graph $\mathcal{G}$, has proven to be a much more efficient alternative for TV. However, it is computationally expensive and the graph $\mathcal{G}$ is kept fixed throughout the algorithm.

Proposed Optimization Problem: We propose to combine the CS setup with a non-local but adaptive and scalable graph TV as follows:

$$\min_x \|Ax - b\|_2^2 + \lambda\|\Phi^*(x)\|_1 + \gamma\|\nabla_{\mathcal{G}}(x)\|_1, \qquad (1)$$

where Φ is the wavelet operator and $\Phi^*(x)$, where $*$ represents the adjoint operation, denotes the wavelet transform of x and $\|\nabla_{\mathcal{G}}(x)\|_1$ denotes the total variation of x w.r.t graph $\mathcal{G}$. The graph $\mathcal{G}$ is constructed initially between the pixels of the vectorized Filtered back Projected (FBP) $x_{fbp} \in \mathbb{R}^{n^2}$ estimate of the sample x, using the standard $\mathcal{K}$-nearest neighbors strategy [3]. The construction of $\mathcal{G}$ is highly scalable due to the use of fast $\mathcal{K}$-nearest neighbor search library (FLANN) [4], which reduces the complexity of graph construction from $\mathcal{O}(n^4)$ (for NLTV) to $\mathcal{O}(n^2\log(n))$ for our method.

Adaptive Algorithm: As x is being refined in every iteration, it is natural to update the graph $\mathcal{G}$ as well in every iteration. This simultaneous update of the graph $\mathcal{G}$ corresponds to the adaptive part of the proposed algorithm and its significance has been explained in detail in the arxived full text [5]. We call our method as 'Adaptive Compressed Sensing and Graph TV' (ACSGT).

The first two terms of the objective function above comprise the *sparse reconstruction* part of our method and model the sparsity of the wavelet coefficients. The second term, to which we refer as the *graph total variation* (GTV) regularizer acts as an additional prior for denoising and smoothing. It can be expanded as:

$$\|\nabla_{\mathcal{G}}(x)\|_1 = \sum_i \|\nabla_{\mathcal{G}} x_i\|_1 = \sum_i\sum_j \sqrt{W_{ij}}\|x_i - x_j\|_1,$$

where W is a sparse weight matrix characterizing the $\mathcal{K}$-nearest neighbors graph $\mathcal{G}$, which is constructed using a Gaussian kernel, i.e, $W_{ij} = \exp(-\|x_i - x_j\|^2/\sigma^2)$. The above expression clearly states that GTV involves the minimization of the sum of the gradients of the signals on the nodes of the graphs. As compared to standard TV, the structure of the sample x is taken into account for reconstruction. It is a well known fact that l_1 norm promotes sparsity, so the GTV can also be viewed as a regularization which promotes sparse graph gradients. This corresponds to enforcing a piecewise smoothness of the signal x w.r.t graph $\mathcal{G}$ [5].

The complete algorithm and various tuning parameters to solve the optimization problem (1) can be found in Algorithm 1 in the full text of this abstract [5].

Results and Conclusions. Fig. 1 shows results and a comparative analysis of reconstructing a 64×64 Shepp-Logan phantom corrupted by 10% Poisson noise, from a sinogram of size 36×95 using various methods. The compared methods include Filtered Back Projection (FBP), statistical reconstruction algorithms such as Compressed Sensing (CS), Compressed Sensing and Total Variation (CSTV) [2], Compressed Sensing and Graph TV (CSGT) which is a non-adaptive version of our algorithm, our proposed adaptive algorithm ACSGT and iterative algorithms such as Kaczmarz method (ART), Randomized Kaczmarz method, Cimmino's method (SIRT) and SART. The quality of reconstructed phantom and intensity plots show that our method out-performs all others because it goes beyond spatial similarity between different regions of an image being reconstructed by establishing a connection between similar regions in the image regardless of spatial distance. For details of experimental setup and other datasets please refer to the full text [5].

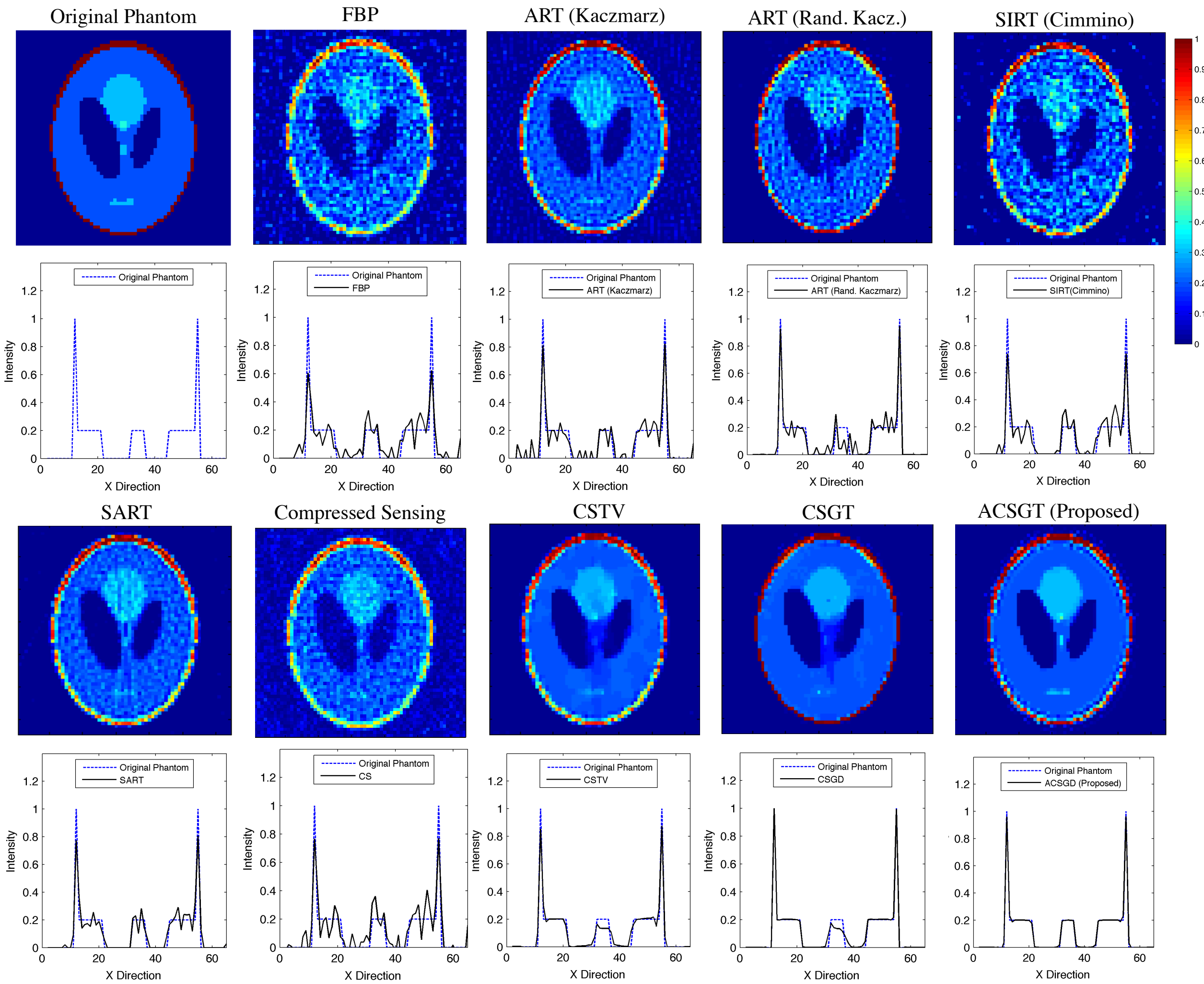

Fig. 1: Comparative analysis of reconstructing Shepp-Logan using various reconstruction methods. The sinogram of a 64×64 Shepp-Logan phantom corrupted with 10% Poisson noise was reconstructed using FBP (Linearly interpolated, Cropped Ram-Lak filter); ART (Kaczmarz/Randomized Kaczmarz, Relaxation Parameter (η) = 0.25, Prior: FBP, Stopping Criteria = 100 iterations); SIRT (Cimmino/SART, (η) = 0.25, Prior: FBP, Stopping Criteria = 100 iterations); CS (500 Iterations, Prior: FBP); CSTV ($\lambda = 0.5$, $\gamma = 0.1$, Prior: FBP, Stopping Criteria = 100 iterations); CSGTV ($\lambda = 0.5$, $\gamma = 0.2$, Prior: Graph from FBP, Stopping Criteria = 100 iterations); ACSGT ($\lambda = 0.5$, $\gamma = 1$, Prior: Patch Graph from FBP updated every iteration, I and J in Algorithm 1 set to 30). ACSGT clearly gives a better intensity profile as compared to all other methods while preserving the edges.

References

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