Joint Sparsity and ”SPID” Calculation of the Stationary Wavelet Transform for Compressed Sensing Reconstruction in Parallel MRI

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***Abstract*—A fast Compressed Sensing (CS) reconstruction scheme for parallel MRI is proposed, utilizing the hybrid “SPID” technique. The proposed method utilizes SPID for calculating the Stationary Wavelet Transform (SWT) of the unknown MR image from the undersampled k-space data; subsequently, it recovers the image through a CS process that promotes joint sparsity of the multi-coils data. In-vivo experiments show that this method eliminates artifacts and expedites the CS convergence.**

I. Introduction

Combining the two powerful frameworks of Compressed Sensing (CS) and parallel MRI (pMRI) has recently emerged as a promising approach for MR image reconstruction and scan time reduction [1]–[3]. However, the practical application of CS-pMRI methods is limited by their long runtimes and heavy computational burden [1]. In some methods, the long runtime may be related to the redundant parallel application of CS to each channel separately, as well as to the low-quality initial guess.

Here, a fast single-image CS-pMRI reconstruction scheme is proposed, utilizing the hybrid pMRI technique ”SPID” (Sensitivity Profile Indexing and Deconvolution) [4]. The proposed method initially calculates the 1D Stationary Wavelet Transform (SWT) of the unknown image using ”SPID”, then produces a high-quality initial guess, and finally recovers the image through a convex optimization process with multi-coil joint sparsity promotion. This method is therefore referred to herein as WaveSPID-CS.

II. Theory

1) Initial guess calculation. SPID is a hybrid non-iterative pMRI technique [4]; uniquely, it receives *sub-sampled k-space data* and calculates the *full xy-domain 1D convolution* between the unknown MR image $f(x,y)$ and a known user-defined kernel $g(x)$, i.e

$$h(x,y) = f(x,y) * g(x). \tag{1}$$

Generally, the SPID method is based on the following process: first, the desired $g(x)$ function is defined by the user; for example, it can be a Gaussian function. Second, a set of weights W is obtained for every x_0 in the image by solving the linear system:

$$g(x-x_0,y) = \sum_{k_x \in K_x} \sum_{i=1}^{N_c} W_{i,k_x}^{x_0} C_i \cdot exp(-jk_x x) \tag{2}$$

where C_i is the sensitivity map of coil i. Finally, the convolution image $h(x,y)$ is calculated pixel-wise by

$$h(x_0,y_0) = \sum_{k_x \in K_x} \sum_{i=1}^{N_c} W_{i,k_x}^{x_0} F^{-1}\{S_i(k_x,k_{y0})\} \tag{3}$$

where $S_i(k_x,k_{y0})$ is the signal acquired by coil i and F is the 1D Fourier transform operator. A more detailed description of SPID can be found in [4].

In the proposed method, SPID is applied twice with two kernels representing the first-level Low-Pass (LP) and High-Pass (HP) SWT decomposition filters of a specific wavelet, e.g. Daubechies-2. This results in the two convolution images:

$$h^{LP}(x,y) = f(x,y) * g^{LP}(x), \quad h^{HP}(x,y) = f(x,y) * g^{HP}(x).$$

Then, $f(x,y)$ is reconstructed according to the wavelet filter bank theory, by simply summing these images [5]:

$$f^{SPID}(x,y) = h^{LP}(x,y) + h^{HP}(x,y). \tag{4}$$

2) CS reconstruction. Subsequently, $f^{SPID}(x,y)$ is used as an initial guess for a CS process that recovers $f(x,y)$ by solving the convex optimization problem

$$minimize_f \|\mathbf{\Psi} f\|_1 \quad s.t. \quad \mathbf{\Phi} C_i f = b_i \quad \forall i = 1,...,N_c \tag{5}$$

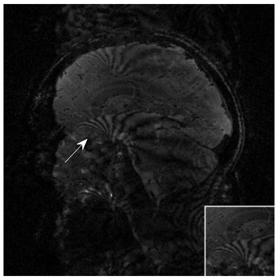
where $\mathbf{\Psi}$ is the SWT operator, $\mathbf{\Phi}$ is an operator representing the 2D Fourier transform and undersampling, and b_i is a vector of coil i samples. Notably, the CS process of eq. (5) promotes joint sparsity of the multi-coils data in the SWT domain; it also recovers a single image rather than a set of coil-specific sensitivity-weighted images.

III. Methods & Materials

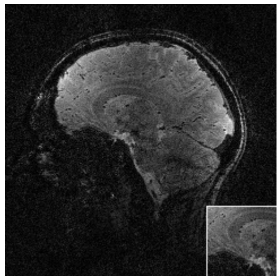
WaveSPID-CS was applied to in-vivo T2* data acquired at 7T using 32 coils. k-Space was undersampled with an acceleration factor of $R = 4$. The SWT filters corresponded to Daubchies-2 wavelets, and eq. (5) was solved using the Projection Onto Convex Setd (POCS) method. For comparison, the same CS-POCS method was also impelemented with a conventional initial guess obtained from zero-flling k-space, hence implementing the method of [6]. Reconstructions were compared to a fully-sampled gold standard by the Normalized Root Mean Square Error (NRMSE) measure.

IV. Results & Discussion

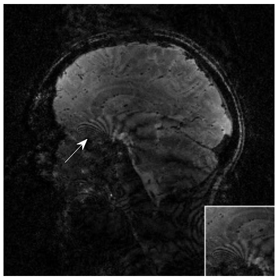
In-vivo results (Fig. 1) show that the aliasing present in the conventional CS initial guess (Fig. 1a) is absent from the proposed WaveSPID-CS initial guess (Fig. 1b). Furthermore, WaveSPID-CS accelerated the CS process: it converged rapidly within 11 iterations only, while the method of [6] converged after 45 iterations (Fig. 2). The proposed method therefor eliminates artifacts and expedites the CS-pMRI reconstruction.



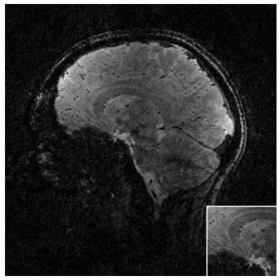
(a) Method [6] initial guess



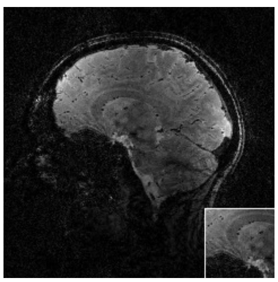
(b) Proposed method initial guess



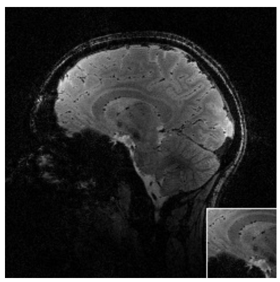
(c) Method [6] after 5 iters.



(d) Proposed method after 5 iters.



(e) Method [6] after 45 iters.



(f) Gold standard (full k-space)

Fig. 1: Reconstruction results for k-space data obtained in a T2* 7T scan, undersampled with a reduction factor of $R = 4$, . Left column (a,c,e): reconstructions obtained using the CS-pMRI method of [6], which includes k-space Zero Filling; arrows point to artifacts. Right column (b,d): reconstructions obtained with the proposed WaveSPID-CS method. (f) Gold standard image calculated from the fully sampled k-space.

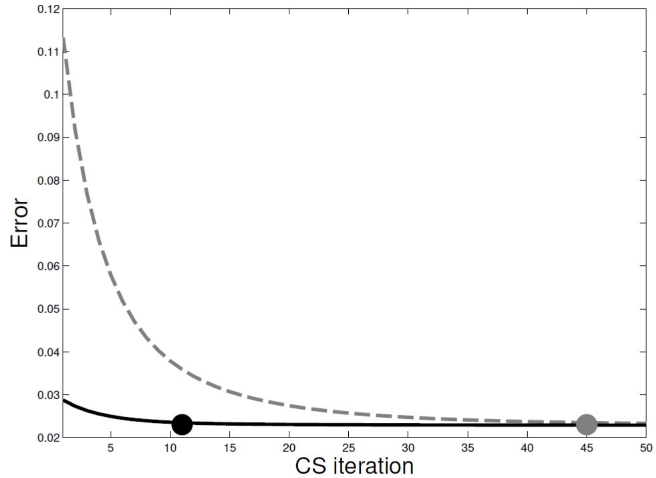


Fig. 2: Reconstruction NRMSE as a function of CS iteration number, for the method of [6] (gray) and the proposed method (black). The circular markers designate the iteration in which each process convergenced. Note the rapid convergence of the proposed method.

References

[1] S. Vasanawala, M. Murphy, M. T. Alley, P. Lai, K. Keutzer, J. M. Pauly, and M. Lustig, "Practical parallel imaging compressed sensing mri: Summary of two years of experience in accelerating body mri of pediatric patients," in *2011 IEEE International Symposium on Biomedical Imaging: From Nano to Macro*. IEEE, 2011, pp. 1039–1043.

[2] R. Otazo, D. Kim, L. Axel, and D. K. Sodickson, "Combination of compressed sensing and parallel imaging for highly accelerated first-pass cardiac perfusion mri," *Magnetic Resonance in Medicine*, vol. 64, no. 3, pp. 767–776, 2010.

[3] D. Liang, B. Liu, J. Wang, and L. Ying, "Accelerating sense using compressed sensing," *Magnetic Resonance in Medicine*, vol. 62, no. 6, pp. 1574–1584, 2009.

[4] H. Azhari, D. K. Sodickson, and R. R. Edelman, "Rapid mr imaging by sensitivity profile indexing and deconvolution reconstruction (SPID)," *Magnetic resonance imaging*, vol. 21, no. 6, pp. 575–584, 2003.

[5] G. Strang and T. Nguyen, *Wavelets and filter banks*. SIAM, 1996.

[6] M. H. Kayvanrad, A. J. McLeod, J. S. Baxter, C. A. McKenzie, and T. M. Peters, "Stationary wavelet transform for under-sampled mri reconstruction," *Magnetic resonance imaging*, vol. 32, no. 10, pp. 1353–1364, 2014.