Joint Multichannel Deconvolution and Blind Source Separation

Ming Jiang
Service d'Astrophysique, CEA Saclay
91191 Gif-sur-Yvette, France
Email: ming.jiang@cea.fr

Jérôme Bobin
Service d'Astrophysique, CEA Saclay
91191 Gif-sur-Yvette, France
Email: jerome.bobin@cea.fr

Jean-Luc Starck
Service d'Astrophysique, CEA Saclay
91191 Gif-sur-Yvette, France
Email: jean-luc.starck@cea.fr

***Abstract*—In the real world, current Blind Source Separation (BSS) methods are limited since extra instrumental effects like blurring have not been taken into account. Therefore, a more rigorous BSS has to be solved jointly with a deconvolution problem, yielding a new inverse problem: deconvolution BSS (DBSS). We introduce an innovative DBSS approach, called DecGMCA, which is based on sparse signal modeling and an efficient alternative projected least square algorithm. Numerical results demonstrate the performance of DecGMCA and highlight the advantage of jointly solving BSS and deconvolution instead of considering these two problems independently.**

I. Introduction

In many multichannel or multi-wavelength imaging applications, it is of great interest to identify and extract the characteristic spectra of sources when they are blindly mixed in the observation. This leads to the study of the Blind Source Separation (BSS) problem. BSS alone is a complex non-convex inverse problem. However, this problem becomes even more challenging when the data are not fully sampled or blurred due to the Point Spread Function (PSF). Therefore, we have to jointly solve both a deconvolution and a BSS problem, yielding a **Deconvolution Blind Source Separation (DBSS)** problem. Suppose we are given $\mathrm{N_c}$ channels of observation $\{\mathbf{y}_i\}_{1\leq i\leq \mathrm{N_c}}$ and each observation is assumed to be a linear mixture of $\mathrm{N_s} \leq \mathrm{N_c}$ sources $\{\mathbf{s}_j\}_{1\leq j\leq \mathrm{N_s}}$. Besides the mixing stage, the observations are degraded by a linear channel-dependent operator $\mathbf{H} = [\mathbf{h}_1^t, \mathbf{h}_2^t, \cdots, \mathbf{h}_{\mathrm{N_c}}^t]^t$, and in most cases this convolutive operator is accessible. More precisely,

$$\forall \nu \in \{1, 2, \cdots, \mathrm{N_c}\},\ \mathbf{y}_\nu = \mathbf{h}_\nu * \left(\sum_{j=1}^{\mathrm{N_s}} A_{\nu,j}\mathbf{s}_j\right) + \mathbf{n}_\nu, \quad (1)$$

where the mixing matrix $\mathbf{A}$ quantifies the contribution of a source in the mixture and $\{\mathbf{n}_\nu\}_{1\leq\nu\leq\mathrm{N_c}}$ denotes the contaminated noise. By applying a Fourier transform, our problem can be more conveniently described in the Fourier domain:

$$\forall \nu \in \{1, 2, \cdots, \mathrm{N_c}\},\ \hat{\mathbf{y}}_\nu = \hat{\mathbf{h}}_\nu \odot \left(\sum_{j=1}^{\mathrm{N_s}} A_{\nu,j}\hat{\mathbf{s}}_j\right) + \hat{\mathbf{n}}_\nu, \quad (2)$$

where $\odot$ denotes the Hadamard product. More precisely, at frequency k of channel ν, the observation $\hat{\mathbf{Y}}$ satisfies:

$$\hat{Y}_{\nu,k} = \hat{H}_{\nu,k}\mathbf{a}_\nu\hat{\mathbf{s}}^k + \hat{N}_{\nu,k}. \quad (3)$$

where $\mathbf{a}_\nu$ denotes a row vector at channel index ν and $\hat{\mathbf{s}}^k$ denotes a column vector of Fourier components of all sources at position k.

II. Algorithm

The GMCA algorithm [1] is an efficient BSS method taking advantage of morphological diversity and sparsity in a transformed space. In the spirit of GMCA, our proposed Deconvolution GMCA (DecGMCA) aims at solving the minimization problem as follows:

$$\min_{\mathbf{S},\mathbf{A}} \frac{1}{2}\sum_{\nu}^{\mathrm{N_c}}\sum_{k}^{\mathrm{N_p}} ||\hat{Y}_{\nu,k} - \hat{H}_{\nu,k}\mathbf{a}_\nu\hat{\mathbf{s}}^k||_2^2 + \sum_{i}^{\mathrm{N_s}}\lambda_i||\mathbf{s}_i\mathbf{\Phi}^t||_0, \quad (4)$$

where $\mathbf{s}_i\mathbf{\Phi}^t$ corresponds the coefficient of the source i in the sparse representation $\mathbf{\Phi}$.

In the spirit of BCR [2], the original problem can be split into two alternating solvable convex sub-problems: estimating $\hat{\mathbf{S}}$ and estimating $\mathbf{A}$, which can be solved by a projected alternating least-squares algorithm. In the estimate of $\hat{\mathbf{S}}$, we propose resorting to a Tikhonov regularization of the least-square estimate in Fourier space to stabilize the multichannel deconvolution step. Then we apply sparse thresholding to retrieve a clean estimate of $\hat{\mathbf{S}}$. This procedure, as it performs regularization in both the Fourier and wavelet spaces, can be interpreted as a multichannel extension of the ForWaRD deconvolution method [3].

Such a projected regularized least-square source estimator is motivated by its lower computational cost. This estimator does not, however, provide an optimal estimate of the sources when the separation process is robust. Thus, in the last step of the algorithm, the estimate of $\hat{\mathbf{S}}$ is properly solved with a minimization method based on the Condat-Vu splitting method ([4], [5]). The full algorithm is presented in alg.1.

III. Experiment

In order to emphasize the advantage of jointly solving both the deconvolution and BSS, we compare our DecGMCA algorithm with a different approach (ForWaRD+GMCA): a channel by channel deconvolution using ForWaRD followed by BSS. Suppose we have three astrophysical sources (left column of fig.1), whose spectra generally respect power law with specific indices. In radio interferometer, the PSF is varying along with channels. We assume the resolution of the PSF decreases linearly with channel index (20 channels in total). The observation, which resides in Fourier space, is not fully sampled, with a percentage of active data 50% (see fig.2). In addition, the Gaussian noise level is fixed to 60 dB.

Having applied DecGMCA, we can see that recovered sources presented in the middle column of fig.1 are well deconvolved and separated. However, if we apply ForWaRD+GMCA, in the right column of fig.1 we observe that the sources cannot be recovered properly and the result is biased. Furthermore, by quantifying errors between estimated sources and ground-truth sources, we can see in tab.I that DecGMCA is very accurate which shows that our estimated sources have a good agreement with the ground-truth. This is owing to the fact that DecGMCA takes into account the entire information of the data, resulting that the relatively bad PSF can be compensated by the relatively good PSF, which is not the case of ForWaRD+GMCA.

IV. Conclusion

We proposed an innovative algorithm Deconvolution GMCA (DecGMCA) to solve the DBSS problems. DecGMCA benefits from sparse signal modeling and a novel projected least-squares algorithm. Numerical experiments show the advantage of the joint resolution of both the deconvolution and unmixing problems.

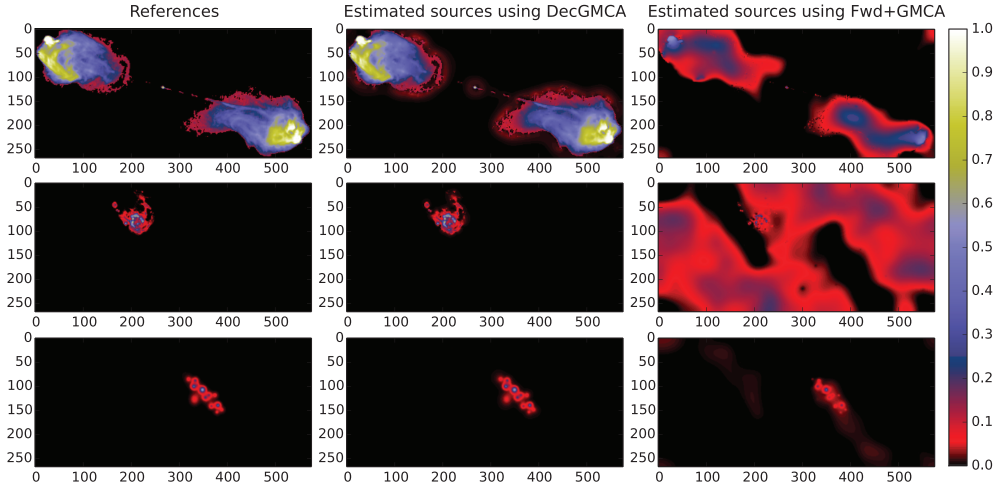

Fig. 1. Illustration of DecGMCA applied on astrophysical images. The raw data is blurred by the masked PSFs and contaminated by the noise: the resolution of the PSF is linearly declined along 20 channels. The ratio between the FWHM of the best PSF and that of the worst PSF is equal to 1/3. PSFs are masked with percentage of active data=50% and the SNR is 60 dB. We apply DecGMCA to separate and recover sources. Left column: Ground-truth of three sources from top to bottom. Middle column: Estimate of three sources by using DecGMCA from top to bottom. Right column: Estimate of three sources by using ForWaRD+GMCA from top to bottom.

Algorithm 1 Deconvolved-GMCA (DecGMCA)

Input: $\hat{\mathbf{Y}}$, $\hat{\mathbf{H}}$, $\lambda^{(0)}$, $\epsilon^{(0)}$
Initialize $\mathbf{A}^{(0)}$
for $i = 1, \dots, \mathrm{N_i}$ **do**
  for $k = 1, \dots, \mathrm{N_p}$ **do**
    $\hat{\mathbf{s}}^k = \left(\sum_{\nu}^{\mathrm{N_c}} \left(\hat{H}_{\nu,k} \mathbf{a}_\nu \right)^t \left(\hat{H}_{\nu,k} \mathbf{a}_\nu \right) + \epsilon' \mathbf{I}_N \right)^{-1} \sum_{\nu}^{\mathrm{N_c}} \hat{H}_{\nu,k} \hat{Y}_{\nu,k} \mathbf{a}_\nu^t$
  end for
  $\mathbf{S}^{(i)} = \mathrm{Re}(\mathrm{FT}^{-1}(\hat{\mathbf{S}}^{(i)}))$
  for $j = 1, \dots, \mathrm{N_s}$ **do**
    $\boldsymbol{\alpha}_j = \mathrm{Th}_{\lambda_j^{(i)}} (\mathbf{s}_j^{(i)} \boldsymbol{\Phi}^t)$
    $\mathbf{s}_j^{(i)} = \boldsymbol{\alpha}_j \boldsymbol{\Phi}$
  end for
  for $\nu = 1, \dots, \mathrm{N_c}$ **do**
    $\mathbf{a}_\nu = \left(\sum_{k}^{\mathrm{N_p}} \hat{H}_{\nu,k} \hat{Y}_{\nu,k} \left(\hat{\mathbf{s}}^k \right)^* \right) \left(\sum_{k}^{\mathrm{N_p}} \left(\hat{H}_{\nu,k} \hat{\mathbf{s}}^k \right) \left(\hat{H}_{\nu,k} \hat{\mathbf{s}}^k \right)^* \right)^{-1}$
  end for
end for
• Last step to ameliorate the estimate $\mathbf{S}$
$\mathbf{S} = \mathrm{argmin}_{\mathbf{S}} \frac{1}{2} \sum_{\nu}^{\mathrm{N_c}} \sum_{k}^{\mathrm{N_p}} ||\hat{Y}_{\nu,k} - \hat{H}_{\nu,k} \mathbf{a}_\nu \hat{\mathbf{s}}^k||_2^2 + \sum_{i}^{\mathrm{N_s}} \lambda_i ||\mathbf{s}_i \boldsymbol{\Phi}^t||_0$
return $\mathbf{A}, \mathbf{S}$

TABLE I
COMPARISON OF RELATIVE ERRORS BETWEEN DECGMCA AND FORWARD+GMCA

Sources	DecGMCA	ForWaRD+GMCA
1	0.14%	54.74%
2	0.27%	1279.21%
3	0.36%	30.12%

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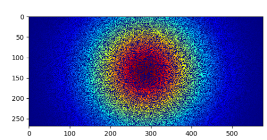
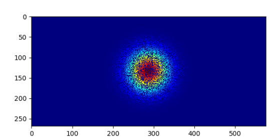


(a) Example of the best resolved PSF over total 20 channels.



(b) Example of the worst resolved PSF over total 20 channels.

Fig. 2. Illustration of masked PSFs (in Fourier space).

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