DOA estimation in fluctuating oceans: put your glasses on!

Angélique Drémeau
ENSTA Bretagne, Lab-STICC (UMR 6285)
Brest, France
Email: angelique.dremeau@ensta-bretagne.fr

Cédric Herzet
INRIA Centre Rennes-Bretagne Atlantique
Rennes, France
Email: cedric.herzet@inria.fr

I. Context and Contributions

In this work, we deal with the problem of estimating the directions of arrival (DOA) of a set of incident plane waves. Most contributions in this field assume that the received signal is only corrupted by some additive noise, see *e.g.,* conventional beamforming (CBF) [1] or MUSIC [2] procedures. Unfortunately, when the waves travel through highly fluctuating media, as in the case of *e.g.,* atmospheric sound propagation [3] or underwater acoustics [4], this model does no longer describe accurately the physics underlying the propagation process. In such cases, a multiplicative phase noise typically corrupts the collected signal, making the corresponding DOA estimation problem much more challenging. In this work, we propose a new methodology to address this issue. Our procedure is grounded on a probabilistic model combining a sparsity-enforcing Bernoulli-Gaussian prior on the DOAs and a Markov-Gaussian model on the phase noise. The estimation of the DOA is based on a mean-field approximation of the Minimum Mean Square Error (MMSE) estimate associated to this probabilistic model.

Our work also relates to the phase retrieval problem (*e.g.,* [5]) where the phase information on the observations is completely missing: only intensities or amplitudes are acquired. Formally, both the phase retrieval problem and the problem considered here share the same observation model but differ in the prior distribution they enforce on the phase noise, the absence of phase information being modeled by a non-informative prior, such as a uniform law (see [6], [7]). We show in the present work how to nicely incorporate fine noise-phase models in this framework, extending in this respect, the approaches proposed in [6], [7].

II. Model

We consider the following observation model:

$$\mathbf{y} = \mathbf{P}\mathbf{D}\mathbf{z} + \boldsymbol{\omega}, \tag{1}$$

where $\boldsymbol{\omega} \in \mathbb{C}^N$ and $\mathbf{P} = \mathrm{diag}(\{e^{j\theta_n}\}_{n=1}^N) \in \mathbb{C}^{N\times N}$ play respectively the role of an additive and a multiplicative phase noise. Matrix $\mathbf{D} = [\mathbf{d}_1 \dots \mathbf{d}_M] \in \mathbb{C}^{N\times M}$ is made up of the steering vectors $\mathbf{d}_i \triangleq [e^{j\frac{2\pi}{\lambda}\Delta\sin(\phi_i)} \dots e^{j\frac{2\pi}{\lambda}\Delta N\sin(\phi_i)}]^T$, where ϕ_i's are some possible angles of arrival, Δ is the distance between two adjacent sensors, and λ is the wavelength of the propagation waves.

With this formulation, the DOA estimation problem is basically equivalent to identifying the positions of the non-zero coefficients in $\mathbf{z}$. When $\mathbf{P}$ is equal to the identity matrix (corresponding to the standard DOA estimation problem), this can been carried out with standard sparse-representation algorithms, as considered in [8], [9]. Here, we consider the more complex case where the phases θ_n's are unknown and obey the Markov model $p(\boldsymbol{\theta}) = \prod_{n=2}^N p(\theta_n|\theta_{n-1})\ p(\theta_1)$, with $p(\theta_n|\theta_{n-1}) = \mathcal{N}(a\,\theta_{n-1}, \sigma_\theta^2)$, $\forall n \in \{2,\dots,M\}$, $a \in \mathbb{R}_+$, and $p(\theta_1) = \mathcal{N}(0, \sigma_1^2)$. From a practical point of view, this model allows us to describe spatial fluctuations of the propagation medium all along the antenna.

To account for the sparsity of $\mathbf{z}$, we consider a Bernoulli-Gaussian (BG) model, which has been now largely used in the sparsity literature (see *e.g.,* [10], [11]). Finally, we classically assume $p(\boldsymbol{\omega}) = \mathcal{CN}(0, \sigma^2\mathbf{I}_M)$.

III. Phase-aware SoBaP

Based on this probabilistic model, we propose to look for the solution of the following MMSE problem

$$\hat{\mathbf{z}} = \arg\min_{\tilde{\mathbf{z}}} \mathrm{E}_{\mathbf{z}|\mathbf{y}}\left[\ \|\mathbf{z} - \tilde{\mathbf{z}}\|_2^2\right],$$

relying on the marginal posterior distribution $p(\mathbf{z}|\mathbf{y}) = \int_{\boldsymbol{\theta}} p(\mathbf{z}, \boldsymbol{\theta}|\mathbf{y})\ d\boldsymbol{\theta}$. The computation of this marginalization being an intractable problem, we propose to resort to a particularization of the "variational Bayes EM algorithm" [12] based on the mean-field approximation $q(\mathbf{z}, \boldsymbol{\theta}) = q(\boldsymbol{\theta}) \prod_i q(z_i)$ of the joint distribution.

Due to space limitation, we omit here the mathematical derivations of this iterative approach, but we refer the reader to our technical report [13]. In the sequel, the proposed algorithm will be dubbed "paVBEM" for "phase-aware VBEM algorithm".

IV. Proof of concept

We consider the problem of identifying the directions of arrival of 2 plane waves from $N = 256$ observations. We assume that the angles of the 2 incident waves can be written as $\phi_k = -\frac{\pi}{2} + i_k\frac{\pi}{50}$ with $i_k \in [1, 50]$, $\forall k \in \{1, 2\}$. The set of angles $\{\phi_i = -\pi + i\frac{\pi}{50}\}_{i\in\{1,\dots,50\}}$ together with the choice of the parameter $\Delta/\lambda = 4$ define the columns of the dictionary $\mathbf{D}$. We set the following parameters for the phase Markov model: $\sigma_1^2 = 10^6$, $\sigma_\theta^2 = 1$ and $a = 0.8$. This corresponds to the situation where one has a large uncertainty on the initial value of the phase noise but connections exist between the phase noise on adjacent sensors.

As a figure of merit, we consider the normalized correlation between the ground truth $\mathbf{z}$ and its reconstruction $\hat{\mathbf{z}}$, *i.e.,* $\frac{|\mathbf{z}^H\hat{\mathbf{z}}|}{\|\mathbf{z}\|_2\|\hat{\mathbf{z}}\|_2}$. This quantity is averaged over 50 realizations for each point of simulation. Figure 1 presents the performance of the following algorithms: *i)* the standard CBF [1]; *ii)* the *prVBEM* algorithm introduced in [7] as a solution to the phase retrieval problem; *iii)* the *paVBEM* procedure proposed in this paper (*BG paVBEM*); *iv)* a relaxed version of *paVBEM* in which the BG prior on $\mathbf{z}$ is replaced by a Gaussian one (*Gaussian paVBEM*). We see that CBF fails to cope with the presence of fluctuations in the phase $\boldsymbol{\theta}$. The performance of the three other algorithms directly relates to the level of information they exploit: the proposed methodology outperforms its relaxed counterpart which, in turn, leads to better performance than the procedure proposed in [7]. This good behavior tends to prove a successful inclusion of the priors. Future work will include further assessment in underwater acoustics.

Acknowledgment

This work has been supported by the DGA/MRIS.

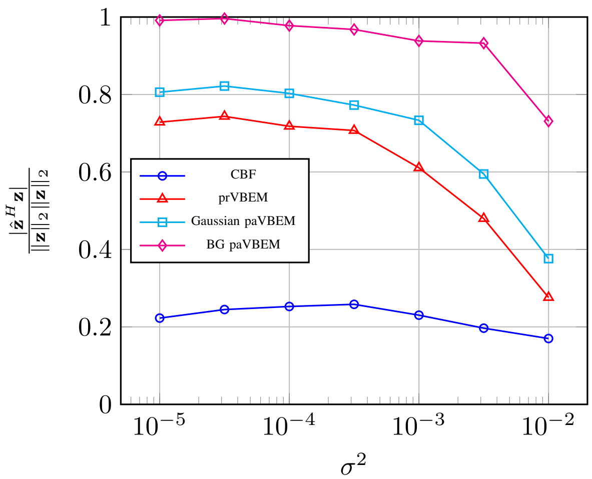

Fig. 1. Evolution of the (averaged) normalized correlation as a function of the variance σ^2.

References

[1] D. H. Johnson and D. E. Dudgeon, *Array signal processing: concepts and techniques*, Englewood Cliffs, NJ, 1993.

[2] R. O. Schmidt, "Multiple emitter location and signal parameter estimation," *IEEE Trans. Antennas Propagation*, vol. AP-34, pp. 276–280, 1986.

[3] S. Cheinet, L. Ehrhardt, D. Juvé, and P. Blanc-Benon, "Unified modeling of turbulence effects on sound propagation," *Journal of Acoustical Society of America*, vol. 132, no. 4, pp. 2198–2209, 2012.

[4] R. Dashen, S.M. Flatté, W.H. Munk, K.M. Watson, and F. Zachariasen, *Sound Transmission Through a Fluctuating Ocean*, Cambridge Monographs on Mechanics. Cambridge University Press, 2010.

[5] E. J. Candès, T. Strohmer, and V. Voroninski, "Phaselift: Exact and stable signal recovery from magnitude measurements via convex programming," *Communications on Pure and Applied Mathematics*, vol. 66, no. 8, pp. 1241–1274, 2013.

[6] P. Schniter and S. Rangan, "Compressive phase retrieval via generalized approximate message passing," in *Communication, Control, and Computing (Allerton)*, October 2012.

[7] A. Drémeau and F. Krzakala, "Phase recovery from a bayesian point of view: the variational approach," in *Proc. IEEE Int'l Conference on Acoustics, Speech and Signal Processing (ICASSP)*, Brisbane, Australia, April 2015, pp. 3661–3665.

[8] W. Mantzel, J. Romberg, and K. Sabra, "Compressive matched-field processing," *Journal of Acoustical Society of America*, vol. 132, no. 1, pp. 90–102, 2012.

[9] A. Xenaki, P. Gerstoft, and K. Mosegaard, "Compressive beamforming," *Journal of Acoustical Society of America*, vol. 136, no. 1, pp. 260–271, 2014.

[10] A. Drémeau, C. Herzet, and L. Daudet, "Boltzmann machine and mean-field approximation for structured sparse decompositions," *IEEE Trans. On Signal Processing*, vol. 60, no. 7, pp. 3425–3438, July 2012.

[11] F. Krzakala, M. Mezard, F. Sausset, Y. F. Sun, and L. Zdeborova, "Statistical physics-based reconstruction in compressed sensing," *Physical Review X*, vol. 2, no. 021005, May 2012.

[12] M. J. Beal and Z. Ghahramani, "The variational bayesian EM algorithm for incomplete data: with application to scoring graphical model structures," *Bayesian Statistics*, vol. 7, pp. 453–463, 2003.

[13] A. Drémeau and C. Herzet, "DOA estimation in structured phase-noisy environments: technical report," Tech. Rep., available at http://arxiv.org:1609.03503, September 2016.