PATCH-BASED INTERFEROMETRIC PHASE ESTIMATION VIA MIXTURE OF GAUSSIAN DENSITY MODELLING IN THE COMPLEX DOMAIN

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ABSTRACT

This paper addresses interferometric phase (InPhase) image denoising; that is, the denoising of phase modulo-2π images from sinusoidal 2π-periodic and noisy observations. The wrapping discontinuities present in the InPhase images, which are to be preserved carefully, make InPhase denoising a challenging inverse problem. We tackle this problem by exploiting the self-similarity of the InPhase images. We propose a novel approach to address the problem by modelling the patches of the phase images using Mixture of Gaussian (MoG) densities in the complex domain. An Expectation Maximization (EM) algorithm is formulated to learn the parameters of the MoG from the noisy data. The learned MoG is used as a prior for estimating the InPhase images from the noisy images using Minimum Mean Square Error (MMSE) estimation. The experiments conducted on simulated and real data of InSAR/InSAS shows results which are competitive with the state-of-the-art techniques

I. INTRODUCTION

Phase imaging systems play a vital role in many present day technologies, namely in the field of surveillance, remote sensing, medical diagnostic, weather forecasting and photography. Often, in such systems, a physical quantity of interest is coded in an image of phase using a suitable coherent imaging techniques (e.g., InSAR, InSAS). Since the phase is closely linked with the wave propagation phenomenon, the measured signals depend only on the principal (wrapped) values of the original phase (absolute phase), which we term as interferometric phase, usually defined in the interval $[-\pi, \pi)$. The interferometric phase is thus a sinusoidal and nonlinear function of the absolute phase, which renders absolute phase estimation a hard inverse problem. In addition, the interferometric phase is usually corrupted by the noise introduced by the acquisition mechanism and electronic equipments, which further complicates the inverse problem which is the inference of the absolute phase from interferomertic measurements. This problem is often tackled in a two-step approach. In the first step, denoising of the noisy wrapped phase is taken care and in the second step, the denoised phase image is unwrapped. InPhase image denoising should be addressed with special care since the wrapping discontinuities should be preserved carefully for the second stage of unwrapping.

The literature shows conventional approaches such as local polynomial approximation [1], time-frequency analysis based filtering [2] to address InPhase image denoising. The state-of-the-art in image restoration focuses on patch-based techniques that exploit non-local self-similarity and sparsity of the natural images like a phase image [3]. The recent work [4], which is representative of the latter approach, adopts a patch-based sparse regression in the complex domain, in which patches are well approximated by a linear combinations of a few atoms taken from a dictionary. Sparse regression identifies low dimensional sub-spaces of clean patches and implicitly projects the noise from a high to low dimensional subspace. This gives rise to noise reduction since the power of the projected noise is proportional to the dimension of the subspace.

In this paper, we propose a novel approach to address the problem of interferometric phase denoising by modelling the patches of complex phase images using MoG densities in the complex domain. Due to the non-local self-similarity of the phase images, the clean patches are well modelled by few eigen-directions of the covariance matrices of the MoG components. In other words, this work exploits the eigenspace based sparsity of the phase patches. The parameters, i.e., the covariance matrix, mean and mixing coefficients of the MoG are learned from complex domain patches of the noisy data. The learned MoG is used as a prior for estimating the interferometric phase images from the noisy images.

The main contribution of our work, which is inspired from the recent state-of-the-art image denoising techniques based on MoGs (see, e.g. [5]), can be summarized as follows: 1) an algorithm to learn the model; this is accomplished by formulating an Expectation Maximization (EM) algorithm for MoG densities in the complex domain; invoking the fact the probability of the patches should be invariant to phase shifts common to all pixels in a patch, we assume circular symmetry to the Gaussian components; 2) Mean Square Error (MMSE) estimates of the clean patches from the noisy

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ones using the learned model.

II. PROBLEM FORMULATION

We assume that the observed data at a given image pixel is

$$z = ae^{j\phi} + n,\ j = \sqrt{-1}, \tag{1}$$

where $a \geq 0$, ϕ, and $n = n_I + jn_Q$ are the values of the amplitude, phase surface, and complex domain noise image at the given pixel. The noise is assumed to be zero-mean Gaussian circular and white with variance σ^2. The denoising strategy is patch-based. We follow an approach similar to that of [4] for the patch formation and aggregation.

II-A. Modelling the patches using MoGs

We model the complex domain patches, $\mathbf{x} \in \mathbb{C}^M$, of the phase image using circularly symmetric MoG, that is,

$$p_X(\mathbf{x}) \triangleq \sum_{i=1}^{K} \alpha_i \mathcal{N}(\mathbf{x}; \mathbf{\Sigma}_i), \tag{2}$$

$$\mathcal{N}(\mathbf{x}; \mathbf{\Sigma}_i) \triangleq \frac{1}{(\pi)^M \det(\mathbf{\Sigma}_i)} e^{-\mathbf{x}^H \mathbf{\Sigma_i}^{-1} \mathbf{x}}, \tag{3}$$

where K is the number of components and $\{\alpha_i, \mathbf{\Sigma_i}\}_{i=1}^K$ are the MoG parameters learned from the noisy data using the EM algorithm.

II-B. Minimum Mean Square Error Estimate

Given the observation model (1) and the MoG (2), we compute the MMSE estimates of the clean patches, $\mathbf{x}$, from the noisy ones, $\mathbf{z}$. After some math, we obtain (see, [5] for a derivation in the real case)

$$\hat{\mathbf{x}}_{\text{mmse}} = \sum_{i=1}^{K} \hat{\boldsymbol{\alpha}}_i \, \hat{\mathbf{x}}_{i,\text{mmse}},$$

where

$$\hat{\boldsymbol{\alpha}}_i = \frac{\alpha_i \mathcal{N}(\mathbf{z}; \mathbf{\Sigma}_i + \sigma^2 \mathbf{I})}{\sum_{k=1}^{K} \alpha_k \mathcal{N}(\mathbf{z}; \mathbf{\Sigma}_k + \sigma^2 \mathbf{I})}$$
$$\hat{\mathbf{x}}_{i,\text{mmse}} = \mathbf{\Sigma}_i (\mathbf{\Sigma}_i + \sigma^2 \mathbf{I})^{-1} \mathbf{z}$$

An illustration of interferometric phase image estimation using a truncated Gaussian surface is shown in Fig.1. Here, the denoising is done using a MoG with 15 components and unwrapping is done using PUMA algorithm [6]. A Peak Signal to Noise Ratio (PSNR) of 39.32dB is obtained from the proposed method where as the PSNR obtained from SpInPHASE [4] for the identical experiment is 38.78dB.

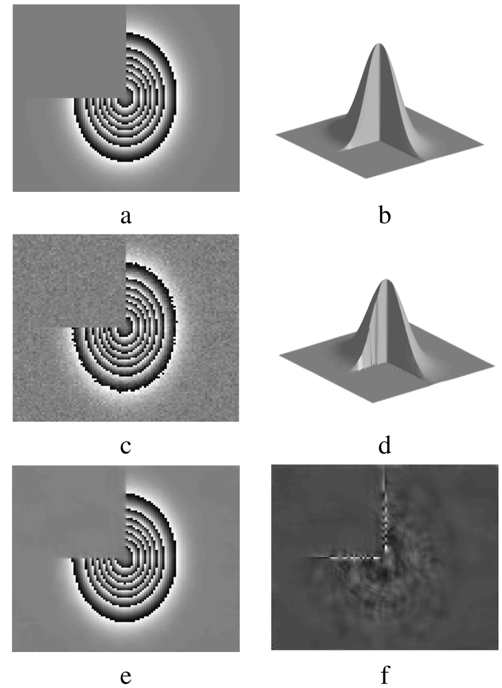

Fig. 1. Experimental results: (a) Original wrapped phase, (b) Original surface, (c) Noisy wrapped phase ($\sigma = 0.5$), (d) Estimate surface, (e) Denoised wrapped phase, (f) Wrapped phase estimate error.

III. CONCLUSION

This paper introduced an effective algorithm for interferometric phase image denoising by modelling the phase patches using MoG densitites in the complex domain. The experiments conducted on real and simulated data show results which are competitive with the state-of-the-art techniques. One of the relevant contributions of our work is that it opens the door to the exploitation of "learned priors" from the specified classes of interferometric phase images, which can then be used in various phase inverse problems.

IV. REFERENCES

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