Multiview Attenuation Computation and Correction

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***Abstract*—Measuring attenuation coefficients is a fundamental problem that can be solved with diverse techniques such as X-ray or optical tomography and lidar. We propose a novel technique based on the observation of a sample from a few different angles. In principle, it can be used in existing devices such as lidar or various types of fluorescence microscopes. It is based on the resolution of a nonlinear inverse problem. We propose a specific computational approach to solve it and show the well-foundedness of the approach on simulated data. Some of the tools developed are of independent interest. In particular we propose new robust solvers for the lidar equation and an adaptation of the nonlocal-means to a specific type of heteroscedastic noise. This can be used to correct attenuation defects in images.**

I. Introduction

Assume that two measured signals u_1 and u_2 are formed according to the following model:

$$u_1(x) = \beta(x) \exp\left(-\int_0^x \alpha(t)\, dt\right) \text{ for } x \in [0,1] \quad (1)$$

$$u_2(x) = \beta(x) \exp\left(-\int_1^x \alpha(t)\, dt\right) \text{ for } x \in [0,1]. \quad (2)$$

The function $\beta : [0,1] \to \mathbb{R}_+$ will be referred to as a density, while $\alpha : [0,1] \to \mathbb{R}_+$ will be called the attenuation map. The signals u_1 and u_2 can be interpreted as measurements of the same scene under opposite directions. The question tackled here is: *can we recover both α and β from the knowledge of u_1 and u_2?*

II. Contributions

a) Applications: The first contribution of this work is to highlight the fact that the above model can be found in many applications ranging from fluorescence microscopy to lidar. Depending on the applications, one may observe more than 2 views, and they may differ by arbitrary angles. To the best of our knowledge, this fact was known in lidar only [1], [2], [3].

b) A Bayesian estimator: It is easy to show that $\alpha(x) = \frac{1}{2}\frac{\partial}{\partial_x}\log\left(\frac{u_2(x)}{u_1(x)}\right)$ and $\beta(x) = \frac{u_1(x)}{\exp\left(-\int_0^x \alpha(t)\, dt\right)}$. Unfortunately, these equations are extremely unstable to noise. Using a maximum a posteriori approach, under a Poisson noise assumption, we construct estimators $\hat{\alpha}$ and $\hat{\beta}$ of α and β defined as the minimizers over $\mathbb{R}^n_+ \times \mathbb{R}^n_+$ of

$$\begin{aligned} F(\alpha,\beta) = \sum_{i=1}^{n}\sum_{j=1}^{2} & [\exp(-(A_j\alpha)[i])\beta[i] \\ & + u_j[i]((A_j\alpha)[i] - \log(\beta[i]))] + R_\alpha(\alpha) + R_\beta(\beta). \end{aligned} \quad (3)$$

The terms $R_\alpha(\alpha)$ and $R_\beta(\beta)$ correspond to regularizers that may depend on the application.

c) A numerical algorithm: It is natural to set $R_\alpha(\alpha)$ and $R_\beta(\beta)$ as convex functionals to get some hope of finding global minimizers. Function F is then convex in each variable separately, but nonconvex on the product-space $\mathbb{R}^n_+ \times \mathbb{R}^n_+$, making it hard to find global minimizers.

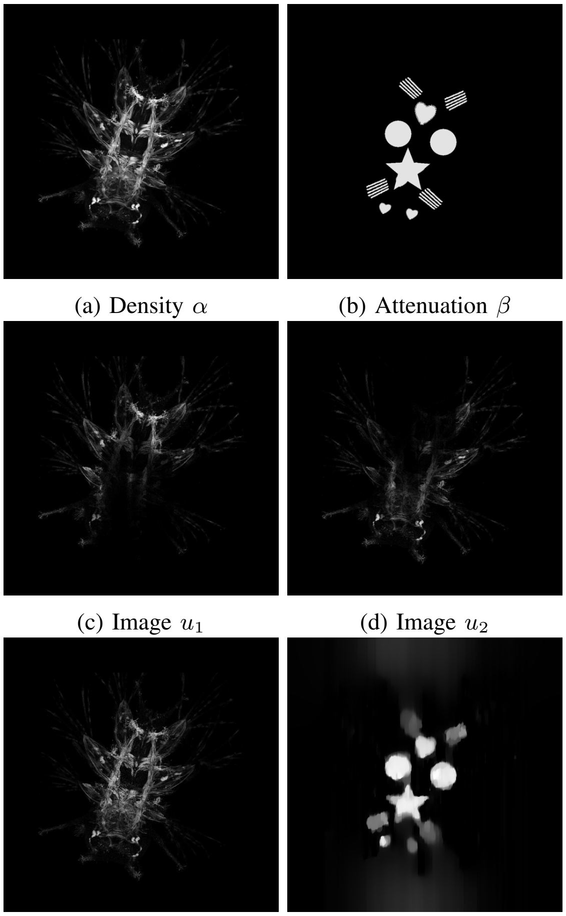
An important observation of this work is that if $R_\beta = 0$ for all β, then the minimizer of $F(\alpha,\beta)$ satisfies $\beta = \frac{u_1+u_2}{\exp(-A_1\alpha)+\exp(-A_2\alpha)}$. Replacing this expression, we obtain a new problem depending on α only which is convex for any convex regularizer R_α:

$$\begin{aligned} \min_{\alpha\, \in \mathbb{R}^n} \sum_{i\, =1}^{n}\sum_{j\, =1}^{2} u_j[i] \Bigg[& (A_j\alpha)[i] \\ & + \log\left(\sum_{j=1}^{2}\exp(-(A_j\alpha)[i])\right)\Bigg] + R_\alpha(\alpha). \end{aligned} \quad (4)$$

This seemingly innocuous problem actually causes many troubles, due to the presence of the logsumexp function and to non common linear integral operators. We develop an efficient numerical strategy to minimize it. This provides a vector α_0 that can be used as a warm start initialization for an alternating algorithm to minimize $F(\alpha,\beta)$. An example of result on a 2D image is shown in Figure (1).

d) Nonlocal means for heteroscedastic noise: The minimization with respect to β of $F(\alpha,\beta)$ corresponds to the problem of restoring a signal attenuated and suffering from Poisson noise. This can be seen equivalently as a problem of denoising with an heteroscedastic (and non orthodox) noise statistics. We develop a statistically sound approach, to extend the NL-means to this setting. An example of result on a 2D image is shown in Figure (2).

III. Typical results



(a) Density α (b) Attenuation β

(c) Image u_1 (d) Image u_2

(e) Estimated density (f) Estimated attenuation

Fig. 1: Illustration of the contribution. A sample (here an insect) has a fluorophore density α shown in Fig. 1a and an attenuation map β shown in Fig.1b. The two measured images u_1 and u_2 are displayed in Fig. 1c and 1d. As can be seen, they are attenuated differently (top to bottom and bottom to top) since the optical path is reversed. From these two images, our algorithm provides a reliable estimate of each map in Fig. 1e and 1f despite Poisson noise.

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References

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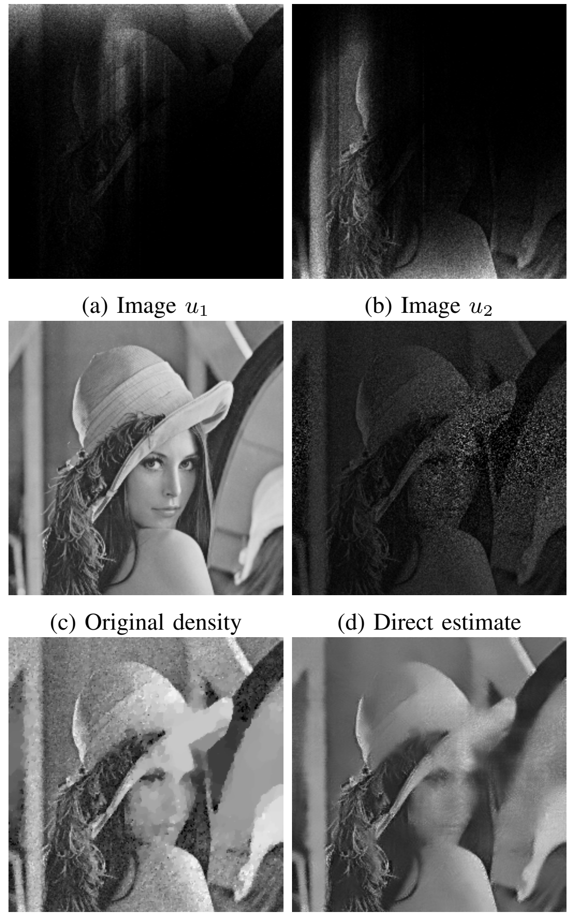

(a) Image u_1 (b) Image u_2

(c) Original density (d) Direct estimate

(e) TV SNR=18.7 (f) NLM SNR=18.8

Fig. 2: In this figure, we compare the efficiency of a taylored total variation minimizer with respect to the proposed nonlocal means algorithm. As can be seen, though the SNR of the two denoised images is similar, the nonlocal means is somewhat more pleasing visually. More importantly, the contrasts of the image are better preserved, which is a key property for an algorithm alternating between a denoising step and an attenuation estimation step.

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