Weighted Diffusion Sparse LMP Algorithm in Non-uniform Noise Environment

Mehdi Korki
School of Software and Electrical Engineering
Swinburne University of Technology, Melbourne, Australia 3122
Email: mkorki@swin.edu.au

Hadi Zayyani
Department of Electrical and Computer Engineering
Qom University of Technology, Qom, Iran
Email: zayyani@qut.ac.ir

***Abstract*—This paper presents an improved version of diffusion least mean p-power (LMP) algorithm for distributed estimation of a sparse parameter vector. We replace the sum of mean square errors with a weighted sum of LMP for global and local cost functions of a network of sensors. The weight coefficients are adaptive and are updated by a simple steepest-descent recursion to minimize the global and local cost functions of the adaptive algorithm. Simulation results show the advantages of the proposed weighted diffusion LMP over the diffusion LMP algorithm specially in the non-uniform noise environments in a sensor network.**

I. Introduction

Distributed estimation is widely used in wireless sensor networks to estimate a parameter vector distributively and cooperatively [1]. Among incremental [1], consensus [1] and diffusion [1], [2]–[4] strategies for distributed estimation, in this paper, we focus on diffusion-based algorithms for the estimation of sparse parameter vectors. A diffusion least mean square (LMS) algorithm has been proposed in [2] and [3]. Moreover, a diffusion least mean p-power (LMP) has been suggested in [5] for distributed estimation in alpha-stable noise environments. Also, a diffusion LMP algorithm with adaptive variable power has been proposed in [6].

In this paper, the global and local cost functions of the diffusion LMP algorithm [5] are defined as the weighted LMP of all the sensor nodes. This is inspired by the non-uniform noise scenarios, where some nodes operate under better noise conditions. It is better to assign more weights to these nodes instead of using uniform distribution of weightings among all nodes. Unlike the diffusion LMP algorithm [5] with constant combination coefficients, for the local cost function, we consider a time varying combination coefficients or a time-varying weight. The weights in the global and local cost functions are updated based on a steepest-descent recursion to minimize the global and local cost functions of the adaptive algorithm.

II. Problem formulation

Consider a sensor network of N nodes distributed over a region. Each sensor at time instant n takes a scalar measurement $d_{k,n}$, which is a linear measurement of a common sparse vector $\boldsymbol{\omega}_o$. The model is $d_{k,n} = \boldsymbol{\omega}_o^T \mathbf{u}_{k,n} + v_{k,n}$, where k is the sensor number, $\mathbf{u}_{k,n}$ is the regression column vector, $v_{k,n}$ denotes the measurement noise and T denotes the transposition. We aim to estimate the common sparse parameter vector $\boldsymbol{\omega}_o$ based on linear measurements $d_{k,n}$ and knowing the regression vectors $\mathbf{u}_{k,n}$. In distributed estimation, we aim to cooperatively estimate the sparse parameter vector $\boldsymbol{\omega}_o$ via in-network processing.

III. The proposed sparse weighted diffusion LMP algorithm

For centralized global estimation of the diffusion LMP algorithm, the sparse parameter vector $\boldsymbol{\omega}_o$ is estimated by minimizing the global cost function [5] $J^{\text{glob}}_{\text{LMP}}(\boldsymbol{\omega}) = \sum_{k=1}^N \mathrm{E}\{|d_{k,n} - \boldsymbol{\omega}^T\mathbf{u}_{k,n}|^p\} + \beta \mathrm{f}(\boldsymbol{\omega})$, where $\mathrm{E}\{.\}$ is the expectation operator, and $\mathrm{f}(\boldsymbol{\omega})$ represents a real-valued convex regularization function weighted by the parameter $\beta > 0$, enforcing sparsity of the solution. Inspired by non-uniform noise conditions and the idea of combination of adaptive filters [7], for weighted diffusion LMP, we propose to use the global cost function $J^{\text{glob}}_{\text{WLMP}}(\boldsymbol{\omega}) = \sum_{k=1}^N \alpha_k(n)\mathrm{E}\{|d_{k,n} - \boldsymbol{\omega}^T\mathbf{u}_{k,n}|^p\} + \beta \mathrm{f}(\boldsymbol{\omega})$, where $\alpha_k(n)$ is the adaptive weights for k'th sensor at time instant n with the constraint $\sum_{k=1}^N \alpha_k(n) = 1$. For the centralized estimation of the unknown sparse parameter vector $\boldsymbol{\omega}$, the steepest-descent recursion is $\boldsymbol{\omega}_n = \boldsymbol{\omega}_{n-1} + \sum_{k=1}^N \mu_k \left(\alpha_k(n)|e_{k,n}|^{p-2}e_{k,n}\mathbf{u}_{k,n} - \beta\zeta(\boldsymbol{\omega})\right)$, where $\zeta(\boldsymbol{\omega}) = \frac{\partial f(\boldsymbol{\omega})}{\partial \boldsymbol{\omega}}$ and $e_{k,n} = d_{k,n} - \boldsymbol{\omega}^T_{k,n}\mathbf{u}_{k,n}$ is the error signal. To update the weight coefficients $\alpha_k(n)$, similar to [7], we assume that $\alpha_k(n) = \frac{\mathrm{e}^{a_k(n)}}{\sum_{j=1}^N \mathrm{e}^{a_j(n)}}$. We update the coefficients $a_k(n)$ by a steepest-descent recursion to minimize $J^{\text{glob}}_{\text{WLMP}}(\boldsymbol{\omega})$, which gives $a_k(n+1) = a_k(n) - \mu_a \alpha_k(n)(|e_{k,n}|^p - \sum_{j=1}^N \alpha_j(n)|e_{j,n}|^p)$. The local cost function at k'th sensor is also defined as $J_k^{\text{loc}}(\boldsymbol{\omega}) = \sum_{l\in\mathbb{N}_k} c_{lk}(n)\mathrm{E}\{|d_{l,n} - \boldsymbol{\omega}^T\mathbf{u}_{l,n}|^p\} + \frac{\beta}{N}\mathrm{f}(\boldsymbol{\omega})$, where c_{lk} is the combination weight from sensor l to sensor k with the constraint $\sum_{l=1}^N c_{lk}(n) = 1$. Similarly, we can assume $c_{lk}(n) = \frac{\mathrm{e}^{a_{lk}(n)}}{\sum_{j=1}^N \mathrm{e}^{a_{jk}(n)}}$. The overall algorithm is a three step algorithm. At the first step, intermediate estimates at each node is calculated by [5], $\boldsymbol{\varphi}_{k,n-1} = \sum_{l\in\mathbb{N}_k} a_{1,lk}(n)\boldsymbol{\omega}_{l,n-1}$,where the coefficients $\{a_{1,lk}\}$ determine which nodes should share their intermediate estimates $\{\boldsymbol{\omega}_{l,n-1}\}$ with node k [5]. At the second step, the nodes update their estimates by [5], $\boldsymbol{\psi}_{k,n} = \boldsymbol{\varphi}_{k,n-1} + \mu_k\alpha_k(n)\sum_{l\in\mathbb{N}_k} c_{lk}(n)|e_{l,n}|^{p-2}e_{l,n}\mathbf{u}_{l,n} - \mu_k\frac{\beta}{N}\zeta(\boldsymbol{\varphi})|_{\boldsymbol{\varphi}=\boldsymbol{\varphi}_{k,n-1}}$. Finally, at the third step, the second combination is performed as [5], $\boldsymbol{\omega}_{k,n} = \sum_{l\in\mathbb{N}_k} a_{2,lk}(n)\boldsymbol{\psi}_{l,n}$, where the coefficients $\{a_{2,lk}\}$ determine which nodes should share their intermediate estimates $\{\boldsymbol{\psi}_{l,n}\}$ with node k [5]. We also assume that all the combination coefficients are equal, i.e. $c_{lk}(n) = a_{1,lk}(n) = a_{2,lk}(n) = \frac{\mathrm{e}^{a_{lk}(n)}}{\sum_{j=1}^N \mathrm{e}^{a_{jk}(n)}}$. To update $c_{lk}(n)$, we use a steepest descent recursion as $a_{lk}(n+1) = a_{lk}(n) - \mu_b\alpha_k(n)\sum_{l\in\mathbb{N}_k} d_{lk}(n)|e_{l,n}|^p$, where $d_{lk}(n) = \frac{\mathrm{e}^{a_{lk}(n)}\sum_{j=1}^N \mathrm{e}^{a_{jk}(n)} - (\mathrm{e}^{a_{lk}(n)})^2}{(\sum_{j=1}^N \mathrm{e}^{a_{jk}(n)})^2}$.

IV. Simulation Results

In our experiment, we consider a distributed network composed of 10 nodes (see Fig. 1). The size of the unknown sparse parameter vector $\boldsymbol{\omega}_o$ is $M = 50$. We consider two scenarios for the measurement noise. First, the measurement noise $v_{k,i}$ is assumed to be Gaussian with zero mean and variance $\sigma^2_{n,i}$. The standard deviation (std) of noise in sensors is assumed to be non-uniform as depicted in Fig. 2. Second, the measurement noise $v_{k,i}$ is assumed to be impulsive, which follows a symmetric alpha-stable distribution with the characteristic function $\varphi(v_{k,i}) = \exp(-\gamma|v_{k,i}|^\alpha)$ [9]. The characteristic exponent $\alpha \in (0, 2]$ controls the impulsiveness of the noise (smaller α leads to more frequent occurrence of impulses) and dispersion $\gamma > 0$ describes the spread of the distribution around its location parameter which is zero for our purposes [9]. The dispersion parameter γ plays a similar role as the variance of Gaussian distribution [5]. We assume non-uniform dispersions for various sensors which are 0.006, 0.001, 0.2, 0.3, 0.002, 0.003, 0.2, 0.5, 0.005, and 0.002 for nodes 1 to 10, respectively. For performance metric, similar to [8], we use mean square deviation (MSD) defined as $\mathrm{MSD}(dB) = 20\log(||\boldsymbol{\omega} - \boldsymbol{\omega}_o||_2)$. Figure 2 shows the standard deviation of noise in various sensors and the final learned weights for the proposed weighted diffusion LMP in Gaussian noise environments. Figure 3 and 4 show MSD curves versus iteration index for 4 different versions of the algorithms in Gaussian and alpha-stable noise environments, respectively. As it is seen in both Fig. 3 and 4, the proposed weighted diffusion LMP algorithms outperform the conventional diffusion LMP algorithms presented in [5].

V. Conclusion

A weighted diffusion LMP algorithm has been proposed for distributed estimation in non-uniform noise environments. Unlike the diffusion LMP algorithm, which utilizes the uniform distribution of weights among sensors, the proposed weighted diffusion LMP algorithm assigns different weights to the sensors with different variances of noise to improve the performance. Compared with the diffusion LMP algorithm, better performance has been achieved for the proposed weighted diffusion LMP algorithm.

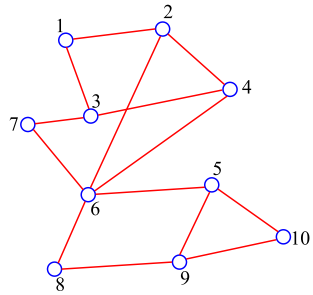

Fig. 1. Topology of the sensor network.

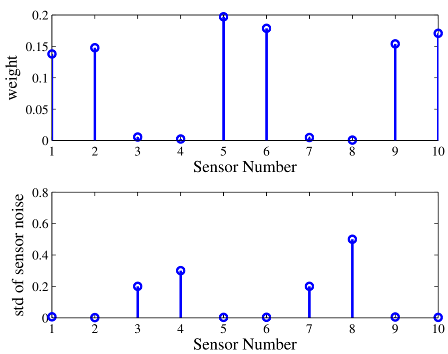

Fig. 2. Non-uniform standard deviation (std) of Gaussian noise in sensors (bottom) and corresponding estimated weights for the proposed weighted diffusion LMP algorithm (top). Note that sensors with higher variance of noise have lower weights and vice versa.

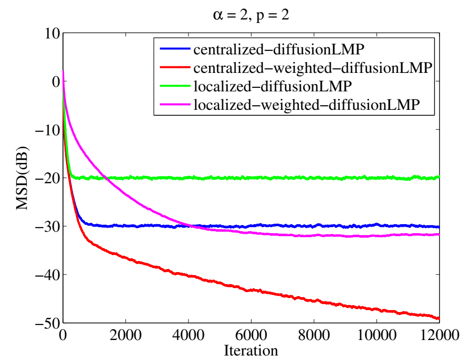

Fig. 3. MSD versus iteration for different versions of diffusion LMP algorithm in Gaussian noise environments, i.e. $\alpha = 2$. The results are averaged over 50 independent trials.

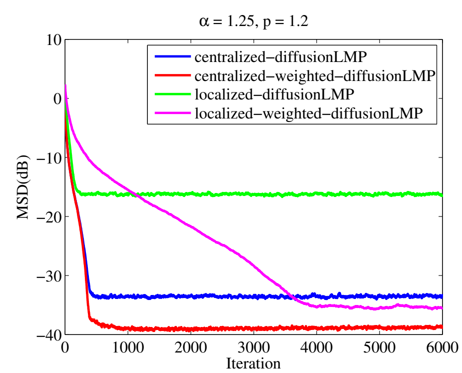

Fig. 4. MSD versus iteration for different versions of diffusion LMP algorithm in alpha-stable noise environments, i.e. $\alpha = 1.25$. The results are averaged over 50 independent trials.

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