Abstract—With general purpose denoisers approaching performance limits, we turn our attention to methods targeted to specific tasks or image classes. Using the recent plug-and-play (PnP) framework, we plug a denoiser based on Gaussian mixture models (GMM) into the iterations of an alternating direction method of multipliers (ADMM), to tackle a data fusion problem known as image sharpening. Results show that our method performs competitively with other state-of-the-art algorithms.

I. INTRODUCTION

In recent years, image denoising has improved only slightly, not due to a lack of research effort, but because we seem to be reaching performance bounds for patch-based general-purpose denoising [2], [5]. Whilst some recent work has been exploiting other approaches, such as deep learning [6], [14], an important question is: how to capitalize on the success of image denoising when addressing other inverse problems (namely deblurring, compressive imaging, and reconstruction)? One answer to this question is the PnP approach [12]. In previous work, we have plugged a GMM-based denoiser [9] in the iterations of an ADMM algorithm [1]. Moreover, instead of a generic GMM prior, we have proposed using priors targeted to specific image classes (e.g., text, faces,...) [10], [11].

In this paper, we address a different inverse problem: hyperspectral sharpening. We are given two types of data: an hyperspectral (HS) cube, with high spectral resolution but low spatial resolution; multispectral (MS) data, with high spatial resolution but low spectral resolution (maybe even a single panchromatic image). The goal is to fuse these data obtaining a cube with high spectral and spatial resolutions. Assuming the images have similar spatial structures, we learn, from the MS data, a GMM adapted to the particular scene at hand and plug it as a denoising prior in an ADMM algorithm that solves the underlying inverse problem.

II. PROBLEM FORMULATION AND METHOD

The hyperspectral sharpening inverse problem can be modeled as

\[ Y_h = ZBM + N_h, \quad Y_m = RZ + N_m, \]

where \( Z \in \mathbb{R}^{L_h \times n_m} \) is the target cube to be estimated, \( Y_h \in \mathbb{R}^{L_h \times n_h} \) is the observed HS data, \( B \in \mathbb{R}^{n_m \times n_h} \) is a spatial convolution operator, \( M \in \mathbb{R}^{n_m \times n_h} \) is a sub-sampling operator, \( Y_m \in \mathbb{R}^{L_m \times n_m} \) is the observed MS data, \( R \in \mathbb{R}^{L_m \times L_m} \) models the spectral responses of the MS sensor, and \( N_h \) and \( N_m \) are Gaussian noises of known variances. Moreover, we assume the sensor has a fixed point spread function, and matrices \( B \) and \( R \) are known.

Typically, an HS cube has hundreds of bands; one way to do dimensionality reduction is to project onto a subspace using the SVD. Then, instead of estimating \( Z \) directly, we estimate latent images \( X \in \mathbb{R}^{p \times n_m} \), and recover \( Z = EX \), where the columns of \( E \in \mathbb{R}^{L_h \times p} \) are the singular vectors corresponding to the \( p \) largest singular values of \( Y_h \). The observation model becomes

\[ Y_h = EZBM + N_h, \quad Y_m = RE + N_m. \]

A classical approach to inverse problem is to compute the maximum a posteriori (MAP) estimate, i.e.,

\[ \hat{X} \in \operatorname{argmin}_X \frac{1}{2} \| EXBM - Y_h \|_F^2 + \lambda \| REX - Y_m \|_F^2 + \tau \phi(X), \]

where \( \phi \) is the negative log-prior (a.k.a. regularizer), while \( \lambda \) and \( \tau \) control the relative weight of each term.

The current tool of choice to solve problems of the form (3) is the ADMM. Given the space limitations, we omit all the steps leading to the final algorithm. In summary: we introduce auxiliary variables, \( V_1, V_2, V_3 \), and constraints \( V_1 = XB, V_2 = X, \) and \( V_3 = X \); the augmented Lagrangian (AL) for the constrained problem includes dual variables \( D_1, D_2, D_3 \) associated to these three constraints; the ADMM algorithm alternates between minimizing the AL w.r.t. each of the primal variables in turn \( X, V_1, V_2, V_3 \) and update the dual variables [1]. The optimization problems for the primal variables are

\[
\begin{align*}
X_{k+1} &= \operatorname{argmin}_X \| XB - V_1 - D_1 \|_F^2 + \| X - V_2 - D_2 \|_F^2 + \| X - V_3 - D_3 \|_F^2, \\
V_1_{k+1} &= \operatorname{argmin}_{V_1} \frac{1}{2} \| EV_1 M - Y_h \|_F^2 + \frac{\mu}{2} \| X - V_1 - D_1 \|_F^2, \\
V_2_{k+1} &= \operatorname{argmin}_{V_2} \frac{1}{2} \| EV_2 M - Y_m \|_F^2 + \frac{\mu}{2} \| X - V_1 - D_2 \|_F^2, \\
V_3_{k+1} &= \operatorname{argmin}_{V_3} \frac{\lambda}{2} \| EV_3 M - Y_m \|_F^2 + \frac{\mu}{2} \| X - V_3 - D_3 \|_F^2,
\end{align*}
\]

where the first three problems are quadratic with closed form solution [8]. The PnP scheme results from noticing that \( V_{k+1} \) can be seen as the solution of a pure denoising problem. In particular, we use a GMM-based MMSE denoiser [9], where the GMM prior is learned from the high spatial resolution MS image, under the hypothesis that the latent image \( X \) has a similar spatial structure.

III. RESULTS

To illustrate the performance of the proposed method, we ran four experiments on a cropped section of the ROSIS Pavia University dataset (messtec.dlr.de/en/technology/dlr-remote-sensing). In all experiments, we start by learning a GMM from the MS data (in this case, a single panchromatic image), and then use it as a prior in the denoising step of ADMM. What differs among the four experiments is the input noise (see Table I) and the target image. In the first two, we sharpen the hyperspectral bands using the panchromatic image (known as PAN-sharpening), while in the last two we use the RGB and near-infrared channels (MS-sharpening). We compare against HySure, using three different quality measures, namely ERGAS, SAM, and SRE [8]. On almost all accounts, the proposed algorithm outperforms HySure, more expressively on MS-sharpening experiments. Figure 1 illustrate the results of experiment 4 in terms of visual quality (in either case, the differences w.r.t. the original image are not noticeable). In all experiments, we considered patches of size 8 by 8, a GMM with 20 components, and the parameters \( \mu, \lambda, \) and \( \tau \) were selected using grid-search.

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Fig. 1: (a) Original HS bands in false color (50,20,5); (b) low-resolution; (c) HySure; (d) proposed.

REFERENCES