Generalized Approximate Message Passing for Noisy Quantized Compressed Sensing

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***Abstract*—Compressed sensing (CS) is a novel technique that allows for stable reconstruction with sampling rate lower than Nyquist rate if the unknown vector is sparse. In many practical applications compressed sensing (CS) measurements are first scalar quantized and later corrupted in different ways. Reconstruction by convetional techniques on such highly distorted measurements will result in poor accuracy. To solve this problem, we use the well known generalized approximate message passing (GAMP) algorithm and tailor it for quantized CS measurements corrupted with noise. We provide the necessary expressions for the nonlinear updates for different noise models, namely symmetric discrete memoryless channel (SDMC) and additive white Gaussian noise (AWGN) channel. Numerical results show superiority of the GAMP algorithm compared to conventional reconstruction algorithms from the literature in both SDMC and AWGN channels.**

***Index Terms*—Generalized approximate message passing, quantization, Bernoulli-Gaussian mixture, compressed sensing**

I. Introduction

In compressed sensing (CS) we take M linear measurements y_i of an N-dimensional K-sparse vector[1] $\mathbf{x}$. Collecting the M (possibly noisy) measurements into the vector $\mathbf{y}$, we get

$$\mathbf{y} = \mathbf{A}\mathbf{x}\,(+\mathbf{w})\,, \tag{1}$$

where $\mathbf{A} \in \mathbb{R}^{M\times N}$ is the measurement matrix with $M < N$, and $\mathbf{w}$ is the noise vector. Even though (1) produces an underdetermined set of linear equations, compressed sensing allows for stable reconstruction if the measurement matrix $\mathbf{A}$ satisfies the Restricted Isometry Property (RIP) [1], [2].

In practice, CS measurements need to be quantized for storage or further digital processing. Quantization inherently introduces additional measurement noise, and a special attention was drawn to combating it's negative effects. Two main approaches are the design of the quantization scheme and the design of the sparse reconstruction algorithm.

If the computational power is not a major restriction, one could apply an Analysis-by-Synthesis (AbS) quantization scheme for CS measurements, where both the codebook and reconstruction algorithm are kept fixed. For a given $\mathbf{y}$, the neighbourhood of the directly quantized $\mathbf{y}$ is explored in hope of finding a better representation of $\mathbf{y}$ with the respect to the codebook and the reconstruction algorithm [3]. Using Bayesian approximate message passing (BAMP) algorithm as the reconstruction algorithm, a significant gain in terms of mean squared error (MSE) could be obtained compared to classical reconstruction algorithms if the correct prior for $\mathbf{x}$ is chosen [4].

Among other algorithms for CS, the generalized approximate message passing (GAMP) [5] algorithm is of particular interest, since it allows to incorporate any knowledge about measurement process as long as it can be written in terms of conditional pdf. It approximates $\mathbb{E}\{\mathbf{x}|\mathbf{y}\}$ using a computationally efficient iterative procedure. The GAMP algorithm for quantized (noiseless) CS was investigated in [6], where the authors show it's superiority against linear minimum mean square error (MMSE) and maximum a posteriori (MAP) reconstruction methods.

A significant disadvantage of the previous work is not investigating the influence of the measurement noise on the MSE. The noise itself may appear for many reasons, including faulty memory, thermal noise at the quantizer, etc.. Another example is transmission of the quantized measurements over a communication channel, in which case the appropriate model is the additive white Gaussian noise (AWGN) channel.

The aim of the paper is to extend the work of [6] and account for different types of channels when designing the corresponding high rate GAMP algorithm.

II. Measurement model

Fig. 1 shows the corresponding transmission chain in the case of the AWGN channel. There, the measurement vector $\mathbf{y}$ can be compactly expressed as

$$\mathbf{y} = \mathcal{Q}(\mathbf{A}\mathbf{x}) + \mathbf{w}\,. \tag{2}$$

In the case of a symmetric discrete memoryless channel (SDMC), each component y_i takes the value $Q(\mathbf{A}_{i,*}\mathbf{x})$ with probabilty $1-p_e$ or a value from the set $\mathcal{I} \setminus Q(\mathbf{A}_{i,*}\mathbf{x})$ with probability $p_e/(2^R-1)$, where $\mathcal{I}$ is the codebook.

III. Results

In the case of an AWGN channel, numerical results presented in Fig. 2 show superior performance of this algorithm in terms of the MSE compared to other algorithms from the literature. The gain can be as large as 10dB for a specific set of parameters. Fig. 3 shows a strong ability of the GAMP algorithm to cope with the symbol errors that occur in a SDMC. The curves in Fig. 4 show that unlike other algorithms, the GAMP algorithm makes use of additional bits per measurement.

[1]A K-sparse vector has at most K nonzero components

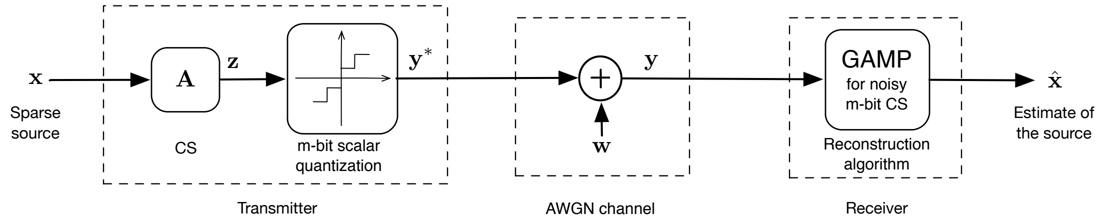

Fig. 1. The signal processing chain. The unknown K sparse vector $\mathbf{x} \in \mathbb{R}^N$ is multiplied with a measurement matrix $\mathbf{A} \in \mathbb{R}^{M \times N}$ to obtain a vector $\mathbf{z} \in \mathbb{R}^M$ of CS measurements. Each component of $\mathbf{y}^*$ represents the quantized version of the respective component in $\mathbf{z}$. Symbols from $\mathbf{y}^*$ are sent through an AWGN channel to obtain the vector of received measurements $\mathbf{y}$.

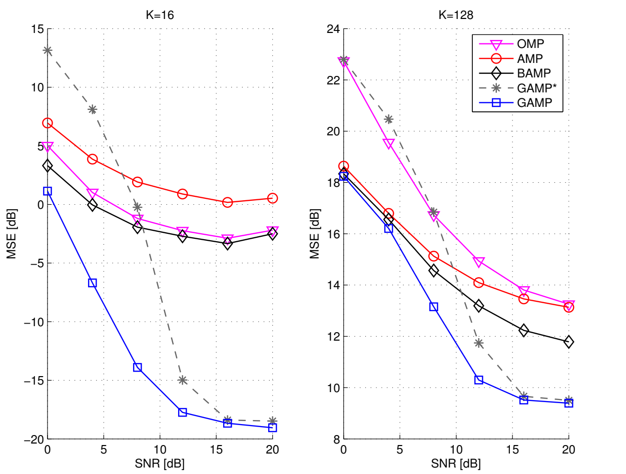

Fig. 2. MSE against SNR for different K in an AWGN channel. GAMP* denotes the GAMP algorithm for noiseless channel model. Parameters $R = 2$, $M = 512$, $N = 512$.

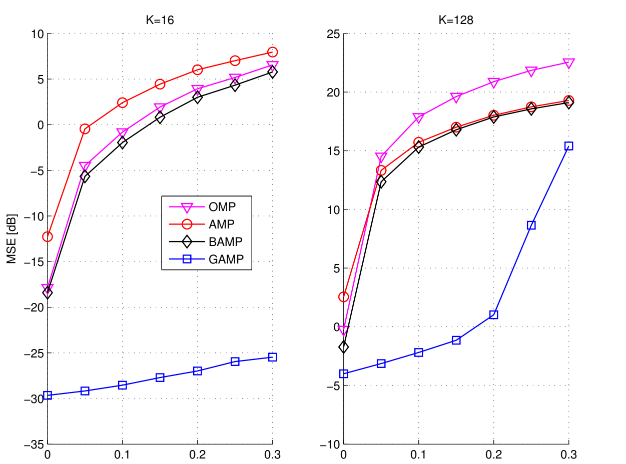

Fig. 3. MSE against the probability of the transmission error p_e for different K in a symmetric channel. Parameters $R = 4$, $M = 512$, and $N = 512$.

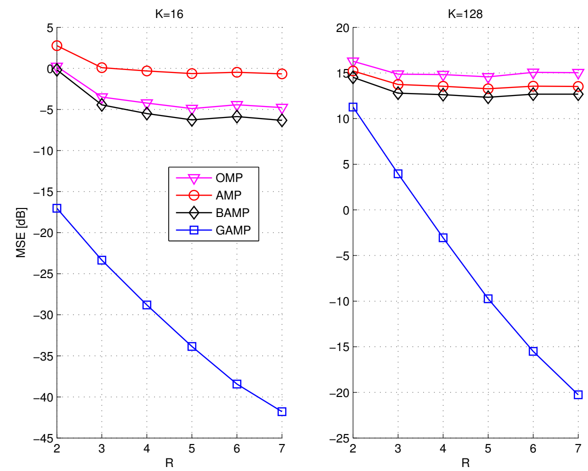

Fig. 4. MSE against R for different K in a symmetric channel. Parameters $p_e = 0.05$, $M = 512$, and $N = 512$.

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