An Efficient Direction-Of-Arrival Estimation Method Based on Weighted Sparse Spectrum Fitting

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***Abstract*—This paper presents a novel Direction-Of-Arrival (DOA) estimation method for Uniform Linear Array (ULA) based on the weighted Sparse Spectrum Fitting (SpSF) which can be regarded as a weighted ℓ_1-regularization problem.**

I. Introduction

Direction-Of-Arrival (DOA) estimation plays an important role in radar, sonar, indoor and outdoor wireless communications [1]–[3]. High resolution DOA estimation methods like MUSIC [4], Root-MUSIC [5], ESPRIT [6] are based on eigenvalue decomposition of sample covariance matrix of array input signal. However, in the case of severe environments like low SNR, small number of snapshots or large number of sources, DOA estimation accuracy often becomes worse due to the lack of effective information [8]–[10].

DOA estimation can also be regarded as a sparse optimization problem to specify DOAs from all the possible directions. The DOA estimation problem is reformulated under the framework of sparse signal reconstruction and can be solved by Sparse Spectrum Fitting (SpSF) [11] with the expression of Lasso compensation [12]. The ℓ_1-regularization problem is often solved by ℓ_1-singular value decomposition (ℓ_1-SVD) [13],[14] which is basically based on the convex optimization [15], [16]. However the regularization process is very sensitive to the penalty term. We recall that the shooting method [17] is computationally efficient and works well for ℓ_1-regularization problem, but it still sensitive to the value of the penalty term.

In this paper, we propose a novel DOA estimation method based on the the weighted ℓ_1-regularization problem. We roughly determine the weight values by the help of Beamformer method, and then tune the DOAs by the weighted ℓ_1-regularization, then the optimization is no longer sensitive to the value of the penalty term.

II. Proposed Approach

Consider the case that L uncorrelated incident waves arrive at K-element ULA of the half-wavelength interelement spacing ($d = \lambda/2$) under an Additive White Gaussian Noise (AWGN) environment. The original SpSF method can be formulated to find an optimum angular spectrum vector $\boldsymbol{p}$:

$$\tilde{\boldsymbol{p}} = \underset{\boldsymbol{p}}{\operatorname{argmin}} \left\{ \|\boldsymbol{r}_x - \boldsymbol{B}\boldsymbol{p}\|_2^2 + \beta \|\boldsymbol{p}\|_1 \right\}, \tag{1}$$

where $\boldsymbol{B} = [\boldsymbol{b}_{11}, \ldots, \boldsymbol{b}_{KK}]$, $\boldsymbol{b}_{kj} = \mathrm{vec}[\boldsymbol{a}(\phi_k)\boldsymbol{a}^H(\phi_j)]$, $\boldsymbol{a}(\phi_k)$ is the array steering vector, $\boldsymbol{r}_x$ is the vectorized input correlation matrix. Here we try to modify the SpSF method so that it does not become sensitive to the penalty term β. Using the angular weight vector $\boldsymbol{w} = [w_1, w_2, \ldots, w_K]$, we have

$$\tilde{\boldsymbol{p}} = \underset{\boldsymbol{p}}{\operatorname{argmin}} \left\{ \|\boldsymbol{r}_x - \boldsymbol{B}\boldsymbol{p}\|_2^2 + \alpha \boldsymbol{w}^T \boldsymbol{p}_a \right\}, \tag{2}$$

where α is the scaling factor, and $\boldsymbol{p}_a = [|p_1|, |p_2|, ..., |p_K|]^T$ is the vector with the absolute values of p_j. Here the term $\boldsymbol{w}^T\boldsymbol{p}_a$ can be regarded as the weighted ℓ_1-norm of $\boldsymbol{p}$.

Based on the above discussion, we arrange the following two-step approach: (a) the first step is a rough estimation stage of non-DOA angles by any simple DOA estimation method just to determine weight values w_j, and (b) the next step is a fine tuning stage by the weighted ℓ_1-norm optimization based on (2). We employ the classical beamformer method as a simple DOA estimation method to be used in the first step. The angular spectrum $\boldsymbol{P}_{bf} = [P_{bf}(\phi_1), P_{bf}(\phi_2), \ldots, P_{bf}(\phi_K)]$ of the beamformer method is given by $P_{bf}(\phi_k) = \boldsymbol{a}^H(\phi_k)\boldsymbol{R}_x\boldsymbol{a}(\phi_k)/\boldsymbol{a}^H(\phi_k)\boldsymbol{a}(\phi_k)$. Note that the angular spectrum $\boldsymbol{P}_{bf}$ becomes small for non-DOA angles. In order to make large values for those angles, the weight w_j is defined as a normalized version of the inverse beamformer spectrum, i.e.,

$$\boldsymbol{w} = P_{\max}\boldsymbol{P}'_{bf}, \tag{3}$$

where $P_{\max} = \max_k(P_{bf}(\phi_k))$, and $\boldsymbol{P}'_{bf} = [1/P_{bf}(\phi_1), 1/P_{bf}(\phi_2), \ldots, 1/P_{bf}(\phi_K)]$. Note that the weight $\boldsymbol{w}$ is normalized by the maximum value of the beamformer spectrum so that the minimum value becomes one.

III. Simulation

The performance of the proposed method is evaluated through some simulation. We try to estimate DOAs in the cases of 300 snapshots, 100 trials, and (1) 4-elements half-wavelength ULA, 2 uncorrelated sources with DOAs from $-10^\circ, 5^\circ$, and (2) 8-elements half-wavelength ULA, 3 uncorrelated sources from $-10^\circ, 0^\circ, 10^\circ$. The DOA estimation performance is evaluated by RMSE (Root Mean Square Error) between the estimated and true DOAs.

Figure 1 shows the example angular spectrums by MUSIC and the proposed methods under the scenario #1 for the cases of (a) 5dB and (b) -15dB. We see from Fig. 1 that the spectrum by the proposed method well separate close-angle waves, especially in low SNR situation in Fig. 1(b). The weighted approach works well even in a severe environment.

To objectively evaluate DOA estimation performance, we compared the averaged RMSE for multiple sources as a function of SNR in low SNR environments from -15dB to 5dB as in Fig. 2. We see from Fig. 2 that the proposed method gives smaller RMSE than the conventional MUSIC and the standard shooting method, especially in the case of low SNR where the RMSE of the conventional methods does not become close to CRLB [18].

IV. Concluding Remarks

This paper investigated the weighted shooting method for high-resolution DOA estimation based on sparse spectrum fitting. The present method achieves smaller RMSE than the conventional approaches in severe environments.

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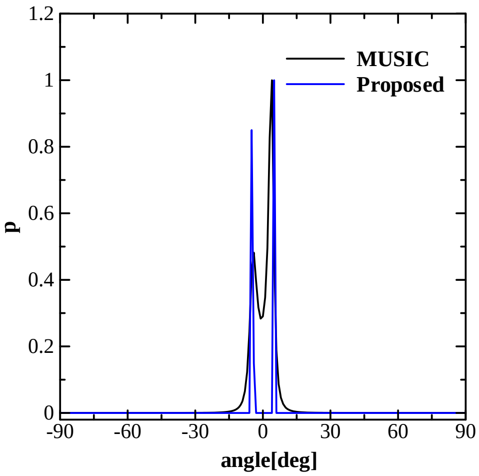
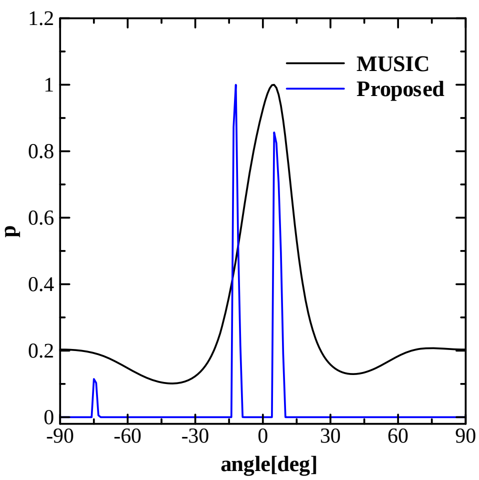
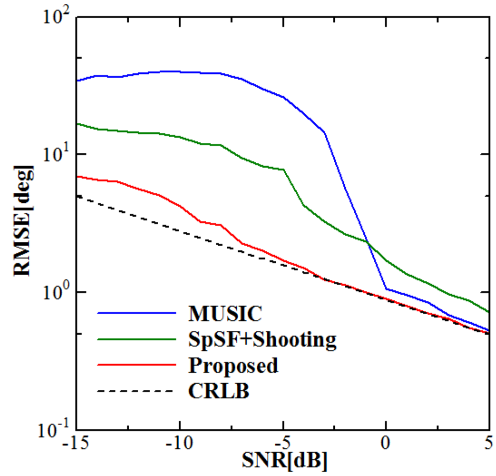


(a) in the case of SNR 5dB

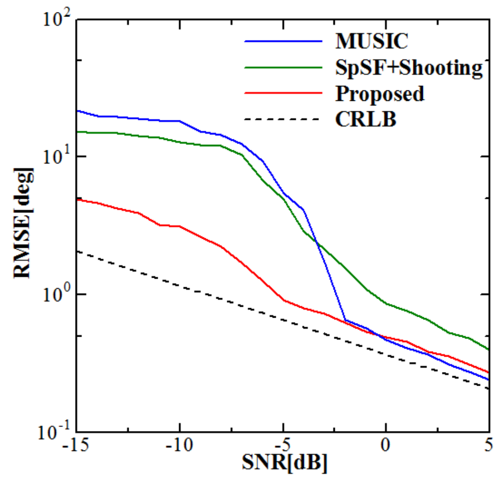


(b) in the case of SNR −15dB

Fig. 1. Comparison of angular spectrums



(a) in the case of scenario #1



(b) in the case of scenario #2

Fig. 2. Behavior of RMSEs as a function of SNR

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