Exploiting Joint Array and Spatial Sparsity for Broadband Source Localisation with Fisher Information Matrix Constraints

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Abstract

Source localisation, such as direction of arrival (DoA) estimation, is an important issue in a number of applications such as underwater acoustic detection, target tracking and environmental monitoring [1] [2]. The key to successful source localisation lies in the reliable extraction of useful information about the states of targets from the observations, which are collected by acoustic transducers, often operated in a noisy environment [2] [3]. Due to the constraints on computation resources, sensing range, communication bandwidth, and energy consumption, it is usually desirable not to use all the sensors in the array to report their measurements at all the time instants [4] [5] [6] [7]. This leads to the problem of sparse array optimisation and sensor selection, which seeks to activate a subset of sensors at different time to optimize the tracking performance under the given constraints.

In this paper, we focus on the problem of using as few sensors as possible to achieve DoA estimation at each sampling time. Traditionally, DoA estimation is addressed by methods, such as Capon beamformer, high-resolution and multiple signal classification (MUSIC) algorithm [8] [9] [10]. Recently, spatial sparsity based optimisation, which aims at extracting meaningful lower-dimensional information from high-dimensional data [11], has attracted great interests. Based on compressive sensing (CS) theory [12], the DoAs can be estimated by solving an optimisation problem constrained with the l_1 norm of a vector of the coefficients corresponding to the source activities in the spatial domain [13], which is assumed to be sparse, implying that only a few sources are active simultaneously. In these existing works, a full array has been used. However, as pointed out earlier, it would be desirable if the spatial sparsity can be used jointly with the array sparsity so that the sources can be localised with as few sensors from the array as possible.

In our previous work [14], we have proposed an optimisation method to exploit jointly the array and spatial sparsity, to achieve source detection in a subset of space using as few sensors as possible at each time instant. The method is operated in a two-step iterative process, where the first step is to find the minimum number of sensors to be used in array and the second step is to perform source localisation based on the least absolute shrinkage and selection operator (LASSO) algorithm using the selected sensors. Both stationary and moving sources are considered. The approach can be initialised at a random location and eventually finds the DoA after it converges. Figure 1 and 2 show the DoA estimations for both stationary source and moving source with noises. Although the results demonstrate satisfatory performances of the joint approach, there are still estimation noise which happen at both ends of the DoA range (+90 degrees, -90 degrees).

To further improve the DoA estimation results, we consider the use of statistical information based on Fisher Information Matrix (FIM) in this joint array and spatial sparsity based optimisation framework. The FIM, which is used to calculate the Maximum Likelihood Estimation (MLE), can be defined as $\mathbb{E}\{(\frac{\partial log\mathbf{y}(\mathbf{x})}{\partial \mathbf{x}})(\frac{\partial log\mathbf{y}(\mathbf{x})}{\partial \mathbf{x}})^T\}$, where y is the received signal and x is the source direction, and $(\cdot)^T$ denotes the transpose of a matrix [15] [16]. The proposed algorithm is still a two-step method.

In the first step, a FIM constraint here is used to limit the difference between the MLE of the received signal $\mathbf{y}$ and the scaled version $diag(\mathbf{w})\mathbf{y}$ where $\mathbf{w}$ is the sparse complex vector weight coefficient for each sensor, so that in the following stage $\mathbf{y}$ can be replaced by $diag(\mathbf{w})\mathbf{y}$ more strictly. The constrained l_1 norm is written as $\|\mathbf{f}(\mathbf{x}) - diag(\mathbf{w})\mathbf{y}\|_1 \leq \beta N$, where $\beta \in \Re^+$ is a threshold to reduce the error before the observed signal is scaled by the weight coefficients, N is the number of potential sensors, $\mathbf{f}(\mathbf{x})$ is the N dimensional vector holding the values of each element in the column of $\mathbf{F}(\mathbf{x})^T$ and the expectation is obtained as the average values.

In the second step, to improve the DoA reconstruction, a constraint is added to the LASSO function as $\|\mathbf{f}(\mathbf{x})^T\mathbf{A} - (\mathbf{A}\mathbf{x})^H\mathbf{A}\|_1 \leq \gamma M$, where the value of FIM is used to constrain the similarity between the DoA reconstruction and the expression of the MLE of $\mathbf{y}$ mapped onto the possible global source directions matrix $\mathbf{A}$, $\gamma \in \Re^+$ is a constrained parameter and M is the number of potential source directions, and $(\cdot)^H$ is a Hermitian operator. As a result, the DoA estimation becomes more robust. Figure 3 and 4 are the results of the narrowband DoA estimations with the FIM constrained joint approach for the moving sources without and with noise. An example of tracking from 50 degrees to -50 degrees is shown to demonstrate the preformance of the proposed joint approach, where 21 of 100 sensors are used at every sampling instant. It can be seen that the estimation errors as happened in the previous results have been reduced.

We also extend this method from the narrowband to the broadband scenario. For the wideband case, the DoA estimation can be done in a similar way for J frequency bands. In the first step, the desired beam response matrix $\mathbf{P} \in \mathbb{C}^{M\times J}$ can be reshaped to an MJ dimensional vector, $\mathbf{p}_{reshape} \in \mathbb{C}^{1\times(MJ)}$. Accordingly, the dictionary matrix $\mathbf{A}$ is modified to $\mathbf{A}_{array} \in \mathbb{C}^{N\times(MJ)}$. In the second step, the dictionary matrix becomes $\mathbf{A}_{spatial} \in \mathbb{C}^{(NJ)\times M}$. Through these reshaping operations, the joint approach can be performed in a narrowband-like manner, with the FIM constraints added into the optimisation process. More results for DoA estimations for both stationary source and moving source, in both noiseless and noisy cases, will be presented in this workshop. A more detailed discussion for using the FIM will be presented, including the process to calculate the MLE via the FIM, the possible convex optimization problem, and the problem of computing the expectation.

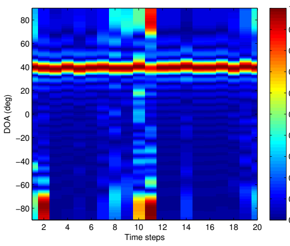

Fig. 1. Narrowband DoA estimations for stationary source (SNR=20dB), 37/100 active sensors.

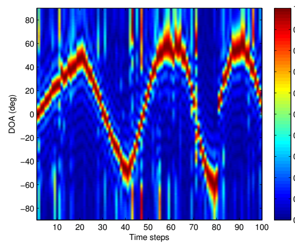

Fig. 2. Narrowband DoA estimations for moving source (SNR=20dB), 22/100 active sensors.

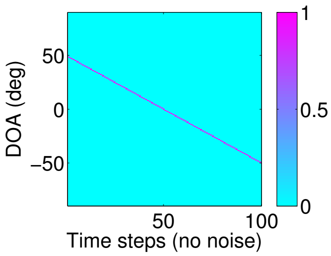

Fig. 3. DoA estimations with FIM constrained joint array sparsity and spatial sparsity based approach for moving source without noise.

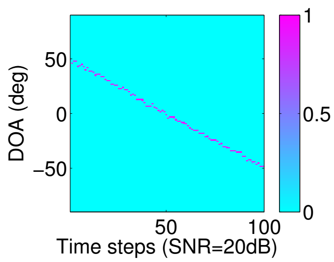

Fig. 4. DoA estimations with FIM constrained joint array sparsity and spatial sparsity based approach for moving source with noise (SNR = 20 dB).

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