

Low-Rank Tensor Regularization for Improved Dynamic Quantitative Magnetic Resonance Imaging

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Abstract—In quantitative MRI, a series of images with different soft-tissue contrast are acquired. For moving organs, such as the heart, this process may be performed across different motion states, leading to ≥ 4 -dimensional images. Furthermore, due to the nature of organ motion and contrast changes, such datasets can be well-represented using low-rank tensors. In this work, we investigate the utility of low-rank tensor regularization for improving the quantification of a state-of-the-art cardiac MRI technique.

I. THE INVERSE PROBLEM IN QUANTITATIVE DYNAMIC MRI

Medical imaging techniques rely on differences in signal intensity (or contrast) between different tissue types. In magnetic resonance imaging (MRI), different contrasts can be generated using the magnetization relaxation properties of tissues. In the past decade, there has been a push for quantitative MRI, where the underlying relaxation properties are quantitatively characterized in a pixel-wise manner, offering robustness independent of the acquisition setup. This requires images of the same anatomy with multiple different contrasts, which leads to long acquisition times, necessitating a trade-off between the spatial or temporal resolutions and SNR of the acquisitions. These challenges are amplified for moving organs, such as the heart.

In this work, we study an image reconstruction problem for a recently proposed dynamic quantitative MRI acquisition of the heart [1]. The goal is to generate a series of images $\mathbf{m}(x, y, t, c)$, where (x, y) is the discrete 2D spatial location, t is the cardiac phase and c is the different contrasts. Once these images are generated, a parametric estimation procedure is performed along the c dimension to generate the quantitative maps of interest for each (x, y) and t [1]. The quality of these maps are quantified by assessing the precision across a homogenous tissue, such as muscle.

II. LOW-RANK TENSOR REGULARIZATION APPROACH

Low-rank matrix regularization has been studied in MRI reconstruction for dynamic or quantitative MRI. However, when both dimensions are varied, the additional benefit of regularization across the multi-dimensional image has been relatively unexplored, except for the use of the Tucker model with pre-selected basis functions [2]. In this work, we propose to use a PARAFAC model for low-rank tensor regularization in a data-adaptive fashion, without requiring basis function pre-estimation from training data.

The acquired measurements are given by

$$\mathbf{y}(t, c) = \mathbf{E}_{t,c}(\mathbf{m}(x, y, t, c)) + \mathbf{n}(t, c), \quad t = 1, \dots, T; c = 1, \dots, C$$

where $\mathbf{E}_{t,c} : \mathbb{C}^{m \times n} \rightarrow \mathbb{C}^p$ is the measurement system, including a partial Fourier matrix and the receiver coil sensitivities, $\mathbf{n}(t, c) \in \mathbb{C}^p$ is measurement noise, t is the cardiac phase, c is the contrast-weighting, and x, y are the discrete spatial locations.

For the 4-dimensional tensor containing $\mathbf{m}(x, y, t, c)$ across all x, y, t, c , we propose to solve the regularized least squares problem:

$$\min_{\{\mathbf{m}(x, y, t, c)\}_{t,c}} \sum_{t,c} \|\mathbf{y}(t, c) - \mathbf{E}_{t,c}(\mathbf{m}(x, y, t, c))\|_2^2$$

subject to $\text{rank}(\mathbf{m}(x, y, t, c)) = F$, (1)

where the tensor rank is defined as the minimum number of rank-one tensors needed to produce \mathbf{m} as their sum [4]. Thus, a rank- F tensor, \mathbf{m} can be written as

$$\mathbf{m}(x, y, t, c) = \sum_{f=1}^F \mathbf{a}_f(x) \mathbf{b}_f(y) \mathbf{c}_f(t) \mathbf{d}_f(c).$$

III. RECONSTRUCTION ALGORITHM AND EXPERIMENTS

The reconstruction algorithm was implemented by alternating between data consistency with the measurements $\{\mathbf{y}(t, c)\}_{t,c}$, and enforcing a low-rank constraint on the 4-dimensional image tensor. The data consistency was implemented as described in [3]. The low-rank tensor constraint in the PARAFAC model was enforced via an alternating least squares approach [4], [5].

Dynamic quantitative MRI of the heart was performed using the technique described in [1] at 3 Tesla field strength. This led to a total of 66 images, with $t = 11$ cardiac phases and $c = 6$ different contrasts as a function of the longitudinal relaxation (T_1) time. Two acquisitions were performed: One at standard resolution ($2 \times 2 \text{ mm}^2$) and one at high resolution ($1 \times 1 \text{ mm}^2$), with 2- and 4-fold undersampling respectively (slice thickness = 10 mm for both cases). Images were reconstructed with and without tensor regularization. Subsequently, quantitative T_1 maps were generated from the reconstructed images, using the parameter estimation approach described in [1]. Precision was defined as the spatial variability across the myocardial (muscle) tissue.

The reconstruction results of individual images for a 4-fold under-sampled high-resolution dataset are shown in Figure 1. Low-rank tensor regularization leads to a visible reduction in noise across different cardiac phases and contrast weightings. The quantitative reconstruction improvement is depicted in Figure 2 for a standard-resolution dataset, with 1.3- to 1.8-fold quantitative precision improvement of T_1 variability for the low-rank tensor regularization approach ($F = 600$) over the conventional parallel imaging method.

In conclusion, the proposed low-rank tensor regularization has the potential to improve undersampled dynamic quantitative MRI reconstruction, offering high-precision quantitative maps at improved spatial resolution within a short acquisition time.

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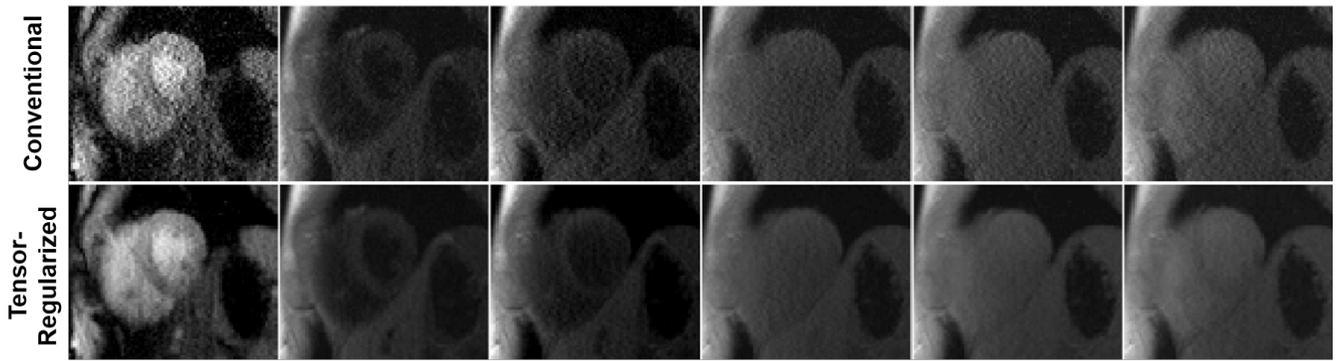


Fig. 1. Comparison of the tensor-regularized reconstruction to the conventional one, for 6 of the 66 images in the image series. For this high-resolution dataset, the noise reduction is visually apparent, while no loss of details is observed on the tissue borders.

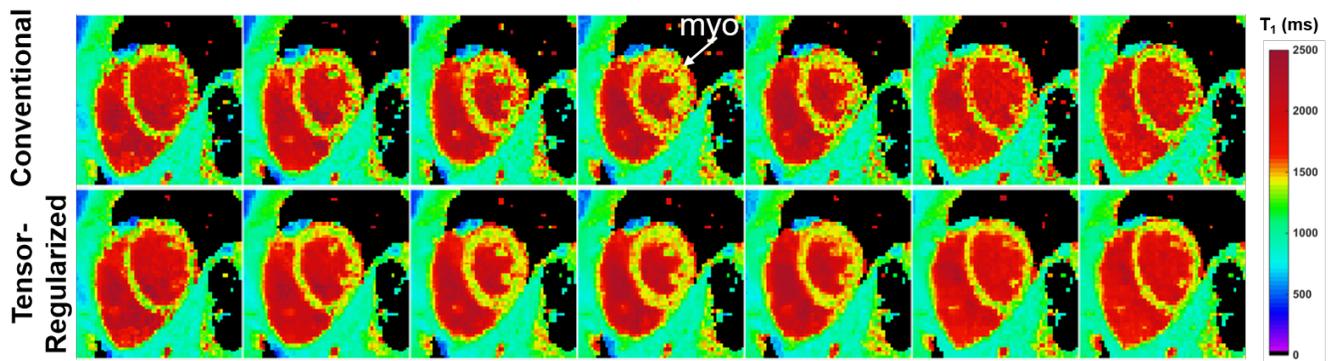


Fig. 2. Parametric longitudinal relaxation (T_1) maps generated following the reconstruction with and without tensor-regularization. 7 of the 11 cardiac phases are displayed. Reduction in the noise variation across the myocardium (labeled “myo”) is visually apparent when comparing the tensor-regularized reconstruction to the conventional one.

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