

# NUWBS: Non-Uniform Wavelet Bandpass Sampling for Compressive RF Feature Acquisition

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**Abstract**—Feature extraction from wideband radio-frequency (RF) signals, such as spectral activity, interferer energy and type, or direction-of-arrival, finds use in a growing number of applications. Compressive sensing (CS)-based analog-to-information (A2I) converters enable the design of inexpensive and energy-efficient wideband RF sensing solutions for such applications. However, most A2I architectures suffer from a variety of real-world impairments. We propose a novel A2I architecture, referred to as non-uniform wavelet bandpass sampling (NUWBS). Our architecture extracts a carefully-tuned subset of wavelet coefficients directly in the RF domain, which mitigates the main issues of most existing A2I converters. We use simulations to show that NUWBS approaches the performance limits of  $\ell_1$ -norm-based sparse signal recovery.

## I. INTRODUCTION

CS-based A2I converters that leverage spectrum sparsity are a promising solution for wideband RF feature acquisition applications [2]–[5]. CS enables the acquisition of larger bandwidths with relaxed sampling-rate requirements, thus enabling inexpensive, faster, and potentially more energy-efficient solutions than traditional Nyquist analog-to-digital converters (ADCs). While a large number of CS-based A2I converters have been proposed in the literature (see, e.g., [2]–[4], [6], [7]), the generally-poor noise performance [8], [9] and sensitivity to real-world hardware impairments prevents their straightforward use in low-power and cost-sensitive applications.

We propose a novel A2I converter for cognitive RF receivers, i.e., radio receivers that are assisted with an A2I converter specifically designed for RF feature extraction. As illustrated in Fig. 1, the A2I converter bypasses conventional RF circuitry and extracts a small set of features directly from the incoming RF signals in the analog domain. The acquired features can then be used by the RF front-end for parameter tuning (e.g., of filters) or by the digital signal processing (DSP) stage. Our approach, referred to as non-uniform wavelet bandpass sampling (NUWBS), combines wavelet pre-processing with non-uniform sampling, which mitigates the key issues of existing A2I solutions, such as signal noise, aliasing, and sensitivity to clock jitter.

## II. NON-UNIFORM WAVELET BANDPASS SAMPLING

Let  $\mathbf{x} \in \mathbb{C}^N$  be a discrete-time,  $N$ -dimensional complex-valued signal vector that we wish to acquire. We assume that the signal  $\mathbf{x}$  has a  $K$ -sparse representation  $\mathbf{s} \in \mathbb{C}^N$ , i.e., the vector  $\mathbf{s}$  has  $K$  dominant non-zero entries in a known (unitary) transform basis  $\Psi \in \mathbb{C}^{N \times N}$  with  $\mathbf{x} = \Psi\mathbf{s}$  and  $\Psi^H\Psi = \mathbf{I}_N$ . In spectrum sensing applications, one typically assumes sparsity in the discrete Fourier transform (DFT) domain, i.e.,  $\Psi = \mathbf{F}^H$  is the  $N$ -dimensional inverse DFT matrix. CS acquires  $M$  compressive measurements as  $y_i = \langle \phi_i, \mathbf{s} \rangle + n_i$  for  $i = 1, 2, \dots, M$ , where  $\phi_i \in \mathbb{C}^N$  are the measurement vectors and  $n_i$  models noise. The measurement process can be written in matrix-vector form as follows:  $\mathbf{y} = \Phi\mathbf{x} + \mathbf{n} = \Theta\mathbf{s} + \mathbf{n}$ . Here, the vector  $\mathbf{y} \in \mathbb{C}^M$  contains all  $M$  compressive measurements, the rows of the

sensing matrix  $\Phi^{M \times N}$  correspond to the measurement vectors  $\phi_i$ ,  $i = 1, 2, \dots, M$ , the  $M \times N$  effective sensing matrix  $\Theta = \Phi\Psi$  models the joint effect of CS and the sparsifying transform, and the vector  $\mathbf{n} \in \mathbb{C}^M$  models noise.

The operating principle of NUWBS is illustrated in Fig. 2.<sup>1</sup> NUWBS first multiplies the input signal  $x(t)$  with a wavelet comb and then, integrates over the support of each wavelet, and subsamples the resulting wavelet coefficients. In discrete time, the sensing matrix  $\Phi$  for NUWBS can be described by taking a small set  $\Omega$  of rows of a (possibly overcomplete) wavelet frame  $\mathbf{W}^H \in \mathbb{C}^{W \times N}$ , where  $\mathbf{W}^H$  contains a specific wavelet on each row and  $W \geq M$  corresponds to the total number of wavelets. Hence, the sensing matrix of NUWBS is  $\Phi = \mathbf{R}_\Omega \mathbf{W}^H$ , where  $\mathbf{R}_\Omega = [\mathbf{I}_N]_{\Omega}$ , is the  $M \times N$  restriction operator that contains of a subset  $\Omega$  the rows of the identity matrix  $\mathbf{I}_N$  and  $M = |\Omega|$  denotes the number of wavelet samples. We can write NUWBS as  $\mathbf{y} = \Theta_{\text{NUWBS}}\mathbf{s} + \mathbf{n}$  with  $\Theta_{\text{NUWBS}} = \mathbf{R}_\Omega \mathbf{W}^H \mathbf{F}^H$ .

It is important to realize that parametrizable wavelets can be generated efficiently in hardware. In particular, we are interested in Gabor or Morlet-like waveforms with a given center frequency, bandwidth, and phase (determined by the sample instant). Each wavelet sample corresponds to point-wise multiplication of the sparse signal spectrum with the bandpass filter equivalent to the Fourier transform of the wavelet. Hence, each wavelet captures a different portion of the sparse spectrum with a different phase and bandwidth. Such wavelets can be generated in hardware by leveraging extensive prior work in the field of ultra-wideband (UWB) impulse technology [10], which allows the generation of wavelet pulses that are widely tunable in frequency, bandwidth, and phase [11].

NUWBS has the following key advantages over non-uniform sampling. First, the analog wavelet transform reduces the bandwidth of the input signal  $x(t)$ , which relaxes the bandwidth of the sample-and-hold (S&H) circuit and the ADC. Second, NUWBS enables full control over a number of parameters, which enables one to tune the wavelets to the signal class to be acquired. See [1] for the details.

## III. PERFORMANCE OF NUWBS

We simulate an empirical phase transition [12], i.e., the rate of correctly recovering the true active frequencies from NUWBS measurements. As a reference, we also include the theoretical phase transition of  $\ell_1$ -norm based signal recovery for a Gaussian measurement ensemble [13]. We use  $N = 256$  frequency bins and generate the signals and NUWBS measurements as detailed in [1]. For support recovery, we use orthogonal matching pursuit [14]. From Fig. 3, we see that NUWBS exhibits similar success and failure rates as predicted by the theoretical phase transition. Hence, NUWBS enables hardware-friendly RF feature extraction while delivering a performance that is close to the theoretical performance limits.

An extended version of this paper has been submitted to the IEEE Transactions on Circuits and Systems – Part I: Regular Papers [1].

<sup>1</sup>While the architecture depicted in Fig. 2 is purely serial, one can deploy multiple parallel branches to further increase the efficacy of NUWBS.

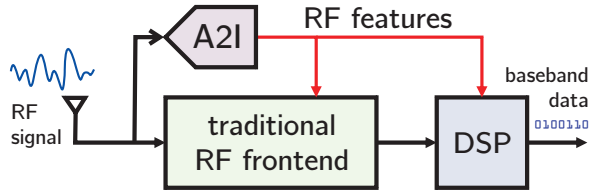


Fig. 1. Overview of a cognitive radio receiver: A traditional RF front-end is enhanced with an A2I converter that extracts RF features directly from the incoming analog RF signals. The A2I converter enables parameter tuning to reduce design margins in the RF circuitry and assists spectrum sensing or awareness tasks in the digital domain.

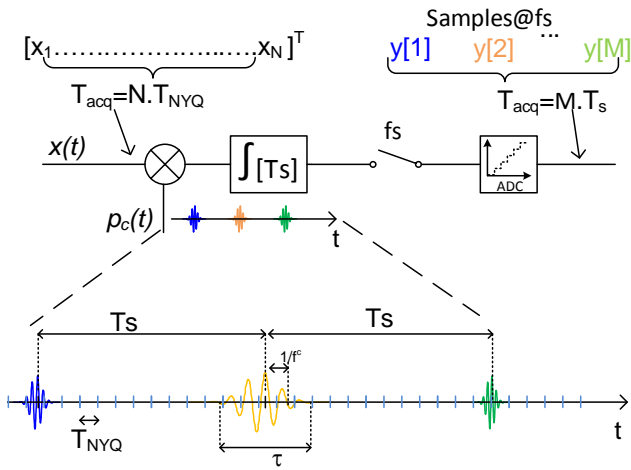


Fig. 2. NUVBS architecture that acquires wavelet samples. NUVBS first multiplies the input signal  $x(t)$  with a wavelet comb  $p_c(t)$  at rate  $1/T_s$  and integrates over each wavelet. One then takes a random subset of wavelet samples and quantizes them using an ADC. Each wavelet is defined by its central frequency  $f^c$  and its width parameter  $\tau$  (the effective pulse duration).

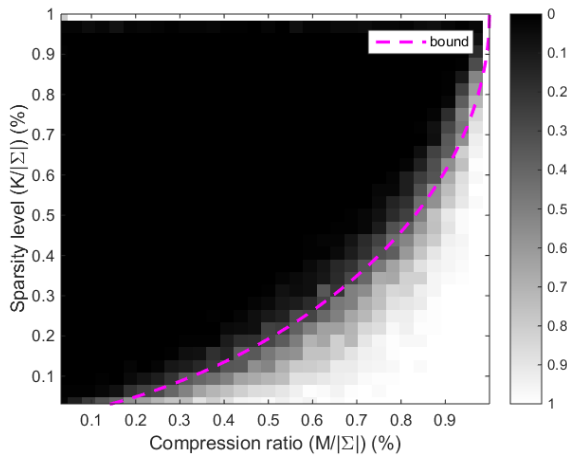


Fig. 3. Empirical phase transition graph of NUVBS for multi-band signal acquisition compared to the theoretical  $\ell_1$ -norm phase transition for a Gaussian measurement ensemble (shown with the dashed purple line). NUVBS exhibits similar performance as the theoretical phase transition.

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